4. (10 points) A car initially traveling 80 ft / sec brakes to a stop in 8 seconds. Its velocity is recorded every 2 seconds and is given in the following table:

t (seconds)	0	2	4	6	8
$v(t)$ (ft/sec)	80	52	28	10	0

(a) Give a good estimate for the distance the car traveled during the course of the 8 seconds. Is your approximation an over or underestimate? How do you know?

Type of sum	**Evaluation**	**Over or underestimate?**
Left sum	$(80)(2) + (52)(2) + (28)(2) + (10)(2) = 340$ ft	Over: velocity is decreasing
Right sum	$(52)(2) + (28)(2) + (10)(2) + (0)(2) = 180$ ft	Under: velocity is decreasing
Average	260 ft	Over: velocity is concave up

(b) To estimate the distance traveled accurate to within 20 feet, how often should the velocity be recorded?

Suppose we record every Δt seconds. Since the velocity is decreasing, the right Riemann sum must be smaller than the distance traveled, which in turn must be smaller than the left Riemann sum. We have

$$L - R = \Big(v(0) - v(8)\Big)\Delta t = 80\Delta t.$$

Therefore if we measure the velocity every $\Delta t = 0.25$ seconds, the left Riemann sum L will be within 20 ft of the actual distance traveled.

(c) Approximate the acceleration of the car 4 seconds after the brakes were applied.

We can approximate this as either $\dfrac{v(4) - v(2)}{4-2} = -12$ ft/s^2, $\dfrac{v(6) - v(4)}{6-4} = -9$ ft/s^2, or as the average of the two (-10.5 ft/s^2).