5. (8 points) A potter has a fixed volume of clay in the form of a cylinder. As he rolls the clay, the length of the cylinder, L, of increases, while the radius, r, decreases. If the length of the cylinder is increasing at a constant rate of 0.2 cm per second, find the rate at which the radius is changing when the radius is 1.5 cm and the length is 4 cm.

[*Recall that the volume of a cylinder of radius r and length L is $\pi r^2 L$.*]

Differentiating the formula for the volume, we find

$$\frac{dV}{dt} = \pi\, 2r \frac{dr}{dt}\, L + \pi\, r^2 \frac{dL}{dt}.$$

Although the shape of the clay is changing, the volume is not, so $\frac{dV}{dt} = 0$. Combining these two statements,

$$\pi\, 2r \frac{dr}{dt}\, L + \pi\, r^2 \frac{dL}{dt} = 0$$

whence

$$\frac{dr}{dt} = -\frac{r\,\frac{dL}{dt}}{2L}.$$

Plugging in the given values $L = 4$, $r = 1.5$, $\frac{dL}{dt} = 0.2$ we deduce

$$\frac{dr}{dt} = -0.0375 \text{ cm / s}.$$