7. (4 points each) Table 1 below displays some values of an invertible, twice-differentiable function $f(x)$, while Figure 2 depicts the graph of the function $g(x)$.

Table 1

x	1	2	3	4	5
$f(x)$	-5	-2	2	4	7
$f'(x)$	5	6	2	3	3
$f''(x)$	1	-1	-3	-2	0

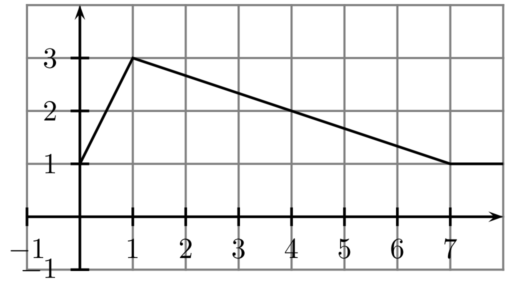

Figure 2: Graph of $g(x)$

Evaluate each of the following. Show your work.

(a) $\displaystyle\int_0^7 g(x)\,dx$

The region under g from 0 to 7 can be viewed as a 1×7 rectangle with a triangle sitting on top of it with base 7 and height 2. Thus,

$$\int_0^7 g(x)\,dx = 7 + \left(\frac{1}{2}\right)(7)(2) = 14$$

(b) $\displaystyle\int_1^3 f'(x)\,dx$

By the fundamental theorem of calculus,

$$\int_1^3 f'(x)\,dx = f(3) - f(1) = 7$$

(c) $\displaystyle\int_1^5 \Big(3f''(x) + 4\Big)\,dx$

$$\begin{aligned}\int_1^5 \Big(3f''(x) + 4\Big)\,dx &= 3\int_1^5 f''(x)\,dx + \int_1^5 4\,dx \\ &= 3\Big(f'(5) - f'(1)\Big) + 4(5-1) \\ &= 10\end{aligned}$$

(d) $\displaystyle\int_1^4 \Big(f'(x)g(x) + f(x)g'(x)\Big)\,dx$

Recognizing $f'(x)g(x) + f(x)g'(x)$ as the derivative of the $f(x)g(x)$ (the product rule!), we apply the fundamental theorem of calculus to find

$$\int_1^4 \Big(f'(x)g(x) + f(x)g'(x)\Big)\,dx = f(x)g(x)\Big|_1^4 = (4)(2) - (-5)(3) = 23$$