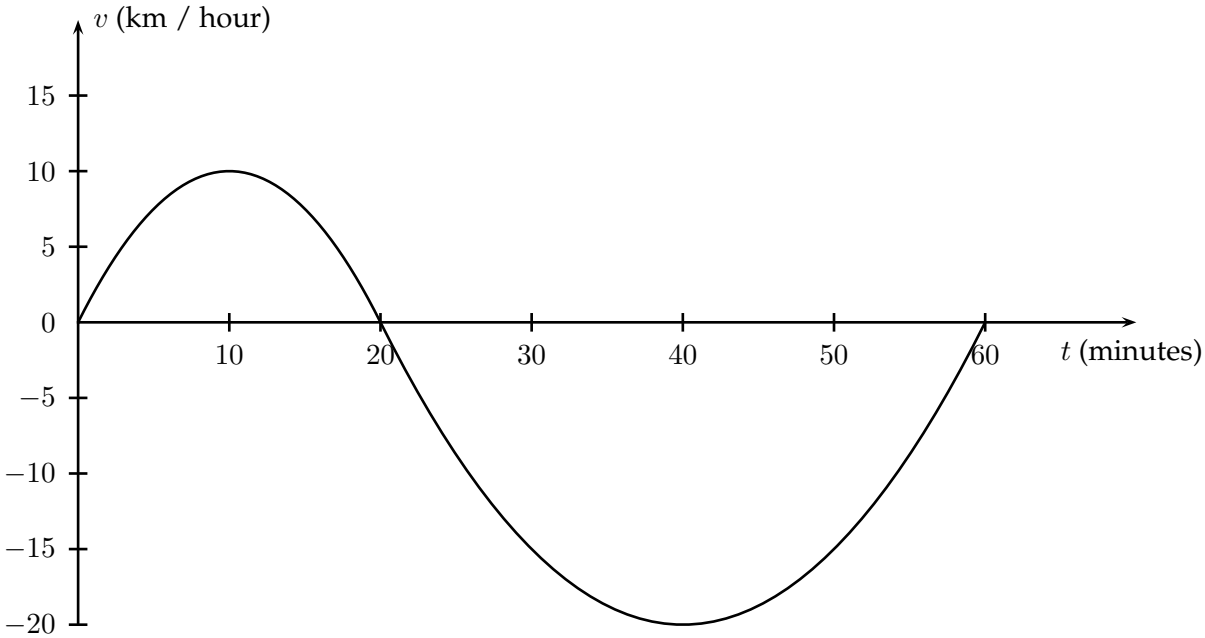
9. A bicyclist is pedaling along a straight road for one hour with a velocity v shown in the figure below. She starts out five kilometers from a lake; positive velocities take her toward the lake.



(a) (2 points) Does the cyclist ever turn around? If so, at what time(s)?

The cyclist turns around 20 minutes into the ride.

(b) (3 points) When is she going the fastest? How fast is she going then? Is she going toward or away from the lake?

40 minutes into the ride, the cyclist is riding away from the lake at a rate of 20 km / hr.

(c) (3 points) When is she closest to the lake? Approximately how close to the lake does she get?

20 minutes into the ride, the cyclist has gone

$$\int_0^{\frac{1}{3}} v(t)\,dt \approx \left(\frac{1}{2}\right)\left(\frac{1}{3}\right)(10) = \frac{5}{3} \text{ km}$$

from her starting position. Since she was traveling towards the lake this whole time, and she started 5 km away from it, she is now about $5 - 1.667 = 3.333$ km from the lake.

(d) (3 points) When is she farthest from the lake? Approximately how far from the lake is she then?

The cyclist is furthest from the lake 1 hour into her ride. From her position 20 minutes into the ride, her position has changed by

$$\int_{\frac{1}{3}}^{1} v(t)\,dt \approx \left(\frac{1}{2}\right)\left(\frac{2}{3}\right)(-20) = -\frac{20}{3} \text{ km.}$$

In part (c) we approximated she was 3.333 km from the lake 20 minutes into the ride; thus she is approximately 6.667 km further away from the lake at the end of the ride, i.e. she is approximately 10 km away at the end of her ride.