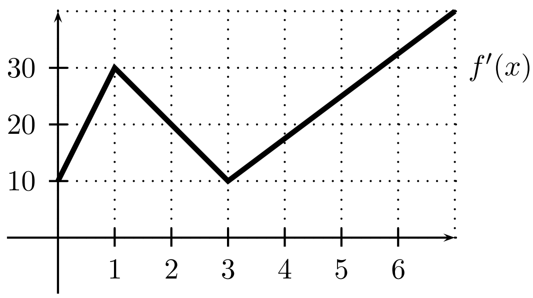
2. [12 points]
Use the graph of the function f' and the table of values for the function g to answer the questions below. Each problem requires only a small amount of work, but you must show it.



x	-20	-10	0	10	20	30
g(x)	0	4	0	-18	-56	-120
g'(x)	6	1	-10	-27	-50	-79

a. [3 points] Write a formula for the local linearization of g near $x = 10$ and use it to approximate $g(10.1)$.

Solution:

$$\begin{aligned} g(x) &\approx g(10) + g'(10)(x-10) \\ g(10.1) &\approx g(10) + g'(10)(10.1-10) \\ &= -18 + (-27)(0.1) = -20.7 \end{aligned}$$

b. [3 points] Using the table, estimate $g''(-10)$.

Solution: Note that $g''(x) \approx \dfrac{\Delta g'(x)}{\Delta x}$. We will use the two x-values closest to -10: $x = -20$ and $x = 0$.

$$g''(10) \approx \frac{g'(0) - g'(-20)}{0-(-20)} = \frac{-10-6}{0-(-20)} = \frac{-16}{20} = \frac{-4}{5} = -0.8$$

c. [3 points] If $f(3) = 30$, find the exact value of $f(1)$.

Solution: By the Fundamental Theorem of Calculus,

$$f(3) - f(1) = \int_1^3 f'(x)dx.$$

This integral represents the area under $f'(x)$ and above the x-axis between $x = 1$ and $x = 3$. Using basic geometry, this area is 40. Thus, $f(3) - f(1) = 40$, so $f(1) = f(3) - 40 = 30 - 40 = -10$.

d. [3 points] Given that $f(3) = 30$, find the exact value of $\int_1^3 g'(f(z))f'(z)dz$.
(Hint: use part (c).)

Solution: We recognize $g'(f(z))f'(z)$ as the derivative of $g(f(z))$, because of the chain rule. By the Fundamental Theorem, the integral above is equal to $g(f(3)) - g(f(1))$, which is equal to $g(30) - g(-10)$, using part c. Filling in values of g from the table, this is $-120 - 4 = -124$.