3. [12 points]
Scott is having a graduation party, and his mom wants to order individual cakes for the guests. Each cake is a right circular cylinder with radius R centimeters, height H centimeters, and volume 250 cubic centimeters. In addition,

- there is a fixed cost of \$3 per cake;
- the entire side of the cake will have maize icing with blue candy "M"s, which costs \$0.02 per square centimeter; and
- the entire top of the cake will have blue icing, which costs \$0.01 per square centimeter.

Recall that the volume of a right circular cylinder with radius R and height H is $V = \pi R^2 H$.

a. [4 points] Find a formula for the cost C of one cake, in terms of its radius R.

Solution:

$$\begin{aligned} C(R) &= 3 + 0.02(\text{area of the side}) + 0.01(\text{area of the top}) \\ &= 3 + 0.02(2\pi RH) + 0.01(\pi R^2) \end{aligned}$$

Solving for H in terms of R in the volume formula, gives $H = 250/\pi R^2$, so

$$\begin{aligned} C(R) &= 3 + 0.02(2\pi R\frac{250}{\pi R^2}) + 0.01\pi R^2 \\ &= = 3 + \frac{10}{R} + 0.01\pi R^2. \end{aligned}$$

b. [8 points] What radius and height should Scott's mom choose for the cakes if she wishes to minimize her costs? What is the minimum price for one cake? (To get credit, you must fully justify your answer using algebraic work.)

Solution: We need to find critical points to find the minimum cost, using the first derivative:

$$C'(R) = \frac{-10}{R^2} + 0.02\pi R.$$

A reasonable domain is $R > 0$, and C' is only undefined when $R = 0$, which is outside the domain. We set $C' = 0$ to find any other critical points.

$$\begin{aligned} \frac{-10}{R^2} + 0.02\pi R &= 0 \\ -10 + 0.02\pi R^3 &= 0 \\ R^3 &= \frac{10}{0.02\pi} \\ R &= \left(\frac{10}{0.02\pi}\right)^{1/3} \approx 5.4193 \end{aligned}$$

Since $C''(R) = \frac{20}{R^3} + 0.02\pi$, we have $C''(5.4193) > 0$, so $R = 5.4193$ is a local minimum. It must also be a global minimum, since C is continuous on the domain and there is only one critical point. Then $H = \frac{250}{\pi 5.4193^2} \approx 2.7096$, and $C = 3 + \frac{10}{5.4193} + 0.01\pi(5.4193)^2 \approx 5.77$.

radius = 5.4193*cm* *height* = 2.7096*cm* *cost* = \$5.77