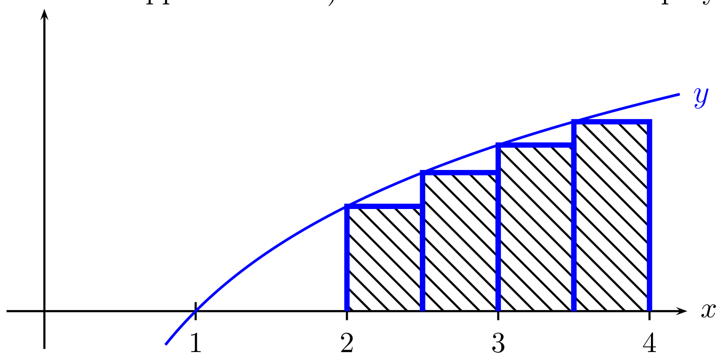
4. [12 points]

a. [6 points] Using 4 equal subdivisions, find a Riemann sum which is an underestimate for

$$\int_2^4 \ln(x)dx.$$

Sketch a graphical representation of your Riemann sum on the axes below, and write "LHS" or "RHS" next to your figure to indicate whether you are using a left-hand sum or a right-hand sum. Write out the terms of the Riemann sum using exact values (no calculator approximations). There is no need to simplify the sum.



LHS

$\int_2^4 \ln(x)dx \approx \underline{0.5\ln 2 + 0.5\ln 2.5 + 0.5\ln 3 + 0.5\ln 3.5}$

b. [3 points]

Show that $\int \ln(x)dx = x\ln(x) - x + C$, where C is a constant.

Solution: We need to check that $\frac{d}{dx}(x\ln(x) - x + C) = \ln(x)$. Using the product rule, we have

$$\frac{d}{dx}(x\ln(x) - x + C) = \left(x \cdot \frac{1}{x} + 1 \cdot \ln x\right) - 1 + 0 = 1 + \ln(x) - 1 = \ln(x).$$

c. [3 points]

Using part (b), find the exact value of the integral $\int_2^4 \ln(x)dx$.

Solution: By the Fundamental Theorem and part (b),

$$\begin{aligned}\int_2^4 \ln(x)dx &= [4\ln(4) - 4 + C] - [2\ln(2) - 2 + C]\\ &= 4\ln 4 - 2\ln 2 - 2\\ &= \ln 4^3 - 2.\end{aligned}$$