4. [12 points]

a. [6 points] Using 4 equal subdivisions, find a Riemann sum which is an underestimate for

\[
\int_{2}^{4} \ln(x)dx.
\]

Sketch a graphical representation of your Riemann sum on the axes below, and write “LHS” or “RHS” next to your figure to indicate whether you are using a left-hand sum or a right-hand sum. Write out the terms of the Riemann sum using exact values (no calculator approximations). There is no need to simplify the sum.

\[
\begin{align*}
\int_{2}^{4} \ln(x)dx & \approx 0.5 \ln 2 + 0.5 \ln 2.5 + 0.5 \ln 3 + 0.5 \ln 3.5 \\
\end{align*}
\]

b. [3 points]

Show that \( \int \ln(x)dx = x \ln(x) - x + C \), where \( C \) is a constant.

**Solution:** We need to check that \( \frac{d}{dx}(x \ln(x) - x + C) = \ln(x) \). Using the product rule, we have

\[
\frac{d}{dx}(x \ln(x) - x + C) = \left( x \cdot \frac{1}{x} + 1 \cdot \ln x \right) - 1 + 0 = 1 + \ln(x) - 1 = \ln(x).
\]

c. [3 points]

Using part (b), find the exact value of the integral \( \int_{2}^{4} \ln(x)dx \).

**Solution:** By the Fundamental Theorem and part (b),

\[
\begin{align*}
\int_{2}^{4} \ln(x)dx &= [4 \ln(4) - 4 + C] - [2 \ln(2) - 2 + C] \\
&= 4 \ln 4 - 2 \ln 2 - 2 \\
&= \ln 4^3 - 2.
\end{align*}
\]