5. [8 points]
Suppose that the *derivative* of a continuous function H is given by the formula

$$H'(t) = \frac{e^{t(t+1)}(t-3)(t+450)}{(4t-100)^{\frac{2}{3}}}.$$

Find all values of t which are critical points of the original function H. Use the first derivative test (and explain your work) to identify each critical point as a local maximum, local minimum, or neither.

Solution: The critical points of H are the values of t for which H' is zero or undefined. Now, $H'(t) = 0$ when $t = 3$ or $t = -450$, and $H'(t)$ is undefined for $t = 25$. Now we need to check whether each of these is a local max, local min, or neither.
These three critical points split the domain into four intervals: $(\infty, -450)$, $(-450, 3)$, $(3, 25)$, and $(25, \infty)$. We will find out the sign of H' for each interval by checking the sign of each factor.

interval	$H'(t)$	H is...
$(\infty, -450)$	$\frac{+\,-\,-}{+} = +$	increasing
$(-450, 3)$	$\frac{+\,-\,+}{+} = -$	decreasing
$(3, 25)$	$\frac{+\,+\,+}{+} = +$	increasing
$(25, \infty)$	$\frac{+\,+\,+}{+} = +$	increasing

Since H is increasing just before $t = -450$ and decreasing right after, $t = 450$ is a local maximum.

Since H is decreasing just before $t = 3$ and increasing right after, $t = 3$ is a local minimum.

Since H is increasing on both sides of $t = 25$, we know $t = 25$ is neither a local max nor a local min.