7. [10 points] Suppose that f is an even function. A portion of f is graphed below.
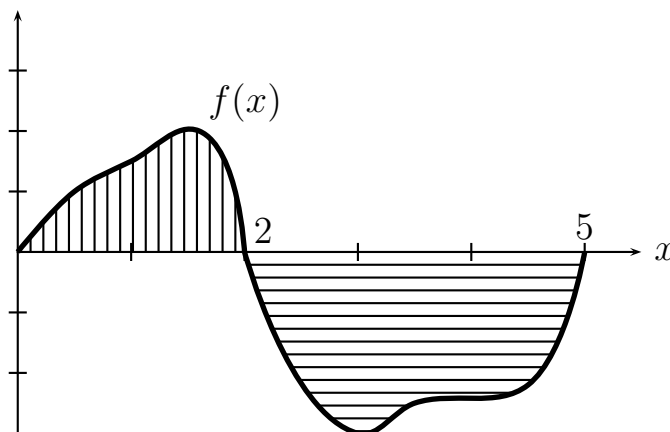

The area of the shaded region between $x = 0$ and $x = 2$ (with vertical stripes) is 3 units, and the area of the shaded region between $x = 2$ and $x = 5$ (with horizontal stripes) is 8 units. Find exact values for each of the following integrals. If it is not possible to find the exact value, write "insufficient information".

a. [2 points] $\displaystyle\int_{-2}^{2} f(x)dx$

Solution: Since f is even, $\displaystyle\int_{-2}^{2} f(x)dx = 2\int_{0}^{2} f(x)dx = 2(3) = 6$.

b. [2 points] $\displaystyle\int_{0}^{5} |f(x)|dx$

Solution: Since we are integrating the absolute value of f, we want the total area between f and the x-axis, between $x = 0$ and $x = 5$, which is $3 + 8 = 11$.

c. [2 points] $\displaystyle\int_{0}^{1} f(2t)dt$

Solution: Since $f(2t)$ is only half as wide as $f(t)$, the shaded area on the left gets compressed to half its width and thus half its area. Thus, $\displaystyle\int_{0}^{1} f(2t)dt = \frac{1}{2}(3) = 1.5$.

d. [2 points] $\displaystyle\int_{5}^{8} f(t - 3)dt$

Solution: The function $f(t - 3)$ is simply $f(t)$ shifted 3 units to the right. Thus, $\displaystyle\int_{5}^{8} f(t - 3)dt = \int_{2}^{5} f(t)dt = -8$.

e. [2 points] $\displaystyle\int_{5}^{2} 9f(z)dz$

Solution: The function $9f(z)$ is 9 times as tall as $f(z)$, so $\displaystyle\int_{5}^{2} 9f(z)dz = 9\int_{5}^{2} f(z)dz = 9 \cdot 8 = 72$.