6. [10 points] The table below gives the expected growth rate, \( g(t) \), in ounces per week, of the weight of a baby in its first 54 weeks of life (which is slightly more than a year). Assume for this problem that \( g(t) \) is a decreasing function.

<table>
<thead>
<tr>
<th>week ( t )</th>
<th>0</th>
<th>9</th>
<th>18</th>
<th>27</th>
<th>36</th>
<th>45</th>
<th>54</th>
</tr>
</thead>
<tbody>
<tr>
<td>growth rate ( g(t) )</td>
<td>6</td>
<td>6</td>
<td>4.5</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>2</td>
</tr>
</tbody>
</table>

**a. [6 points]** Using six subdivisions, find an overestimate and underestimate for the total weight gained by a baby over its first 54 weeks of life.

**Solution:** The gain rate is a decreasing function, so a left-hand sum will be an overestimate and a right-hand sum an underestimate. The left-hand sum is

\[
\int_{0}^{54} g(t) \, dt \approx (6 + 6 + 4.5 + 3 + 3 + 3)(9) = 229.5 \text{ oz},
\]

and the right-hand sum

\[
\int_{0}^{54} g(t) \, dt \approx (6 + 4.5 + 3 + 3 + 3 + 2)(9) = 193.5 \text{ oz}.
\]

That is, we expect the weight gain to be between 12 and 14 lb!

**b. [4 points]** How frequently over the 54 week period would you need the data for \( g(t) \) to be measured to find overestimates and underestimates for the total weight gain over this time period that differ by 0.5 lb (8 oz)?

**Solution:** We know that the difference between the over- and underestimates is over - under = \( |g(54) - g(0)| \Delta t \). Thus we need \( \Delta t \leq 8/(6 - 2) = 2 \) weeks. So we would need data for \( g(t) \) every two weeks.

---