6. [10 points] The table below gives the expected growth rate, $g(t)$, in ounces per week, of the weight of a baby in its first 54 weeks of life (which is slightly more than a year).[1] Assume for this problem that $g(t)$ is a decreasing function.

week t	0	9	18	27	36	45	54
growth rate $g(t)$	6	6	4.5	3	3	3	2

a. [6 points] Using six subdivisions, find an overestimate and underestimate for the total weight gained by a baby over its first 54 weeks of life.

Solution: The gain rate is a decreasing function, so a left-hand sum will be an overestimate and a right-hand sum an underestimate. The left-hand sum is

$$\int_0^{54} g(t)\,dt \approx (6+6+4.5+3+3+3)(9) = 229.5 \quad \text{oz},$$

and the right-hand sum

$$\int_0^{54} g(t)\,dt \approx (6+4.5+3+3+3+2)(9) = 193.5 \quad \text{oz}.$$

That is, we expect the weight gain to be between 12 and 14 lb!

b. [4 points] How frequently over the 54 week period would you need the data for $g(t)$ to be measured to find overestimates and underestimates for the total weight gain over this time period that differ by 0.5 lb (8 oz)?

Solution: We know that the difference between the over- and underestimates is over $-$ under $= |g(54) - g(0)|\Delta t$. Thus we need $\Delta t \leq 8/(6-2) = 2$ weeks. So we would need data for $g(t)$ every two weeks.

[1] Riordan J. *Breastfeeding and Human Lactation*, 3rd ed. Boston: Jones and Bartlett, 2005, p.103, 512-513.