8. [14 points] A car speeds up at a constant rate from 10 to 70 mph over a period of half an hour, between $t = 0$ and $t = 1/2$. Its fuel efficiency, $E(v)$, measured in miles per gallon, depends on its speed, v, measured in miles per hour.

a. [4 points] Write an integral which gives the total distance traveled by the car during the half hour.

Solution: The total distance traveled by the car during the half hour is

$$\int_0^{1/2} v(t)\,dt.$$

b. [5 points] Write an integral which gives the average fuel efficiency of the car during the half hour.

Solution: The average fuel efficiency of the car during the half hour is

$$\frac{1}{60}\int_{10}^{70} E(v)\,dv.$$

Or, in terms of t, the integral

$$\frac{1}{1/2}\int_0^{1/2} E(v(t))\,dt$$

is equivalent.

c. [5 points] For speeds v greater than 70 mph suppose the relationship between E and v is given by

$$E(v) = 2 + v^{-av}$$

for some constant a. Using this formula, write an expression for the definition of the derivative $E'(82)$. Do not evaluate your expression.

Solution: The derivative of E at $v = 82$ is

$$E'(82) = \lim_{h\to 0} \frac{(2 + (82+h)^{-(82+h)a}) - (2 + 82^{-82a})}{h}.$$