5. [12 points] A rectangle has one side on the x-axis and two vertices on the curve

$$y = \frac{36}{9 + x^2}.$$

This curve is graphed below. Find the x- and y- coordinates of all four vertices of the rectangle with largest area. You must show that your vertices maximize the area of the rectangle.