1. [8 points] A ship is sailing out to sea from a dock, moving in a straight line perpendicular to the coast. At the same time, a person is running along the coast toward the dock, hoping desperately to jump aboard the departing ship. Let $b(t)$ denote the distance in feet between the ship and the dock t seconds after its departure, and let $p(t)$ denote the distance in feet between the person and the dock t seconds after the ship's departure. The situation is depicted below for your reference:

Suppose that 10 seconds after the ship's departure, it is 40 feet from the dock and is sailing away at a speed of 20 ft/sec. At the same moment, the person is 30 feet from the dock and running toward it at 14 ft/sec.

a. [2 points] What is $b'(10)$? What is $p'(10)$?

Solution: Since the distance between the ship and the dock is increasing at 20 ft/sec while the distance between the person and the dock is decreasing at 14 ft/sec, we have

$$b'(10) = 20, \text{ and } p'(10) = -14.$$

b. [6 points] Is the distance between the person and the ship increasing or decreasing 10 seconds after the ship's departure? How fast is it increasing or decreasing? (Include units in your answer, and keep in mind that distance is measured along a straight line joining the person and the ship.)

Solution: If the distance between the ship and the person is denoted t seconds after the ship's departure is denoted $d(t)$, then by the Pythagorean Theorem,

$$b(t)^2 + p(t)^2 = d(t)^2.$$

Differentiating this gives:

$$2b(t)b'(t) + 2p(t)p'(t) = 2d(t)d'(t).$$

We know that $b(10) = 40$, $p(10) = 30$, and $d(10) = \sqrt{40^2 + 30^2} = 50$. If we plug this information as well as the numbers from part (a) into the above equation and solve for $d'(t)$, we find

$$d'(t) = \frac{760}{100} = 7.6.$$

Since this is positive, the distance is increasing at 7.6 ft/sec.