3. [16 points] A truck is driving along a straight highway. The function $v(t)$ gives its velocity in m/sec after t seconds on the highway, and the function $a(t)$ gives its acceleration in m/sec^2 after t seconds on the highway. Consider the following table of values for $v(t)$ and $a(t)$, keeping in mind that $a(t) = v'(t)$.

t	0	10	20	30	40
$v(t)$	13	21	27	32	35
$a(t)$	1.0	0.7	0.55	0.4	0.2

In the following questions, **include units** wherever appropriate.

a. [3 points] Use a tangent line approximation to estimate the velocity of the truck after 33 seconds on the highway.

Solution:

$$v(33) \approx v(30) + 3 \cdot v'(30) = 32 + 3 \cdot 0.4 = 33.2 \text{ m/sec}.$$

b. [3 points] Do you expect that your approximation in part (a) is an overestimate or an underestimate? Briefly explain your answer based on the information in the table.

Solution: Since $a(t) = v'(t)$ is decreasing, $v(t)$ is concave down, so the approximation is an overestimate.

c. [4 points] Use a left-hand sum with four subdivisions to approximate the distance traveled by the truck in the first 40 seconds on the highway. Write out each term of the sum as well as the final answer.

Solution:

$$10(13 + 21 + 27 + 32) = 930 \text{ m}.$$

d. [3 points] Do you expect that your approximation in part (c) is an overestimate or an underestimate? Briefly explain your answer based on the information in the table.

Solution: Since $v(t)$ is increasing, the approximation is an underestimate.

e. [3 points] How frequently would velocity measurements need to be made in order to ensure that the left-hand sum and right-hand sum approximating the distance traveled in the first 40 seconds agree to within 1 meter?

Solution: We need

$$1 = (35 - 13)\Delta t,$$

so $\Delta t = 1/22$, or in other words, measurements should be made 22 times per second.