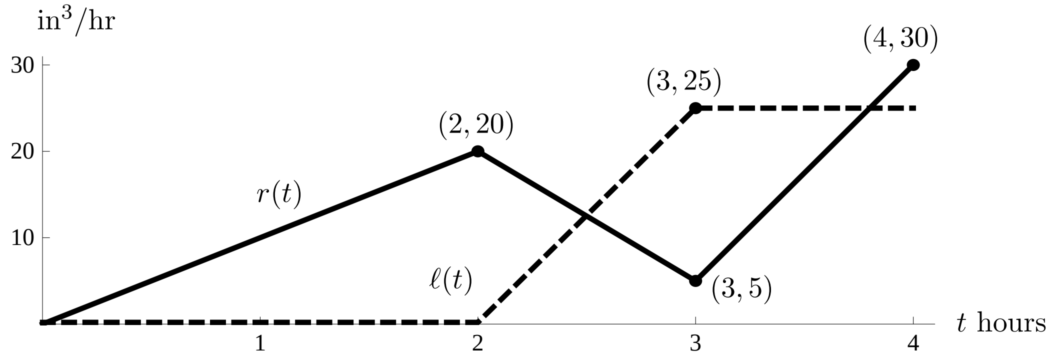
6. [12 points] A large bucket is left outside during a storm, and the bucket begins to fill with rain. The rain starts at midnight, at which point the bucket is empty. At 2am, the bucket springs a leak and some water begins to drip out of it. The function $r(t)$ is the rate at which rain is falling into the bucket t hours after midnight, measured in in^3/hr, while the function $\ell(t)$ is the rate at which water is leaking out of the bucket t hours after midnight, measured in in^3/hr. These functions are graphed below.



Be sure to **include units** in your answers to the following questions. No explanation is necessary, but partial credit may be given for correct work. Assume the bucket is big enough that it never overflows during the storm.

a. [3 points] How much water was in the bucket at 3am?

Solution: The amount of water that has entered is the area under the solid curve from 0 to 3, which is 32.5in^3, and the amount that has exited is the area under the dashed curve from 0 to 3, which is 12.5in^3. Thus, there is 20in^3 of water in the bucket at 3am.

b. [2 points] At what time was the amount of water in the bucket greatest?

Solution: This occurs when $r(t) = \ell(t)$ and $r(t) - \ell(t)$ is going from positive to negative, which is at $t = 2.5$, or 2:30am.

c. [3 points] What is the largest amount of water that was in the bucket between midnight and 4am?

Solution: This is the amount that was in the bucket at 2:30am, which is $\int_0^{2.5}(r(t) - \ell(t))dt = 25\text{in}^3$.

d. [2 points] At what time was the amount of water in the bucket increasing fastest?

Solution: This is when $r(t) - \ell(t)$ is largest, which is at 2am.

e. [2 points] Write an integral expressing the average rate at which rain fell into the bucket over the period from midnight to 4am. You do not need to evaluate your integral.

Solution:

$$\frac{1}{4}\int_0^4 r(t)dt$$