8. [13 points] Use the family of functions of the form

\[ f(x) = ax - \ln(1 + e^{bx}) \]

to answer the following questions. The constants \( a \) and \( b \) are both positive.

a. [4 points] Use the given formula for \( f(x) \) to give an explicit expression for the limit definition of \( f'(x) \). Check your expression carefully, as no partial credit will be given on this part of the problem. Do not evaluate your expression.

\[
\text{Solution: } f'(x) = \lim_{h \to 0} \frac{a(x + h) - \ln(1 + e^{b(x+h)}) - ax + \ln(1 + e^{bx})}{h}.
\]

b. [4 points] Compute \( f'(x) \) using the rules of differentiation. Do not try to evaluate your expression from (a).

\[
\text{Solution: } f'(x) = a - \frac{be^{bx}}{1 + e^{bx}}.
\]

c. [5 points] When \( a < b \), the function \( f(x) \) has a critical point at

\[ x = \frac{1}{b} \ln \left( \frac{a}{b - a} \right). \]

Using the second-derivative test, determine whether this critical point is a local maximum, local minimum, or neither.

\[
\text{Solution: } \text{The second derivative is } f''(x) = -\frac{(1 + e^{bx})b^2e^{bx} - be^{bx}be^{bx}}{(1 + e^{bx})^2} = -\frac{b^2e^{bx}}{(1 + e^{bx})^2}.
\]

This is always negative, so the critical point is a local maximum.