8. [12 points] Mitch puts a thin metal ring in an oven. A picture of the ring, which is made by removing a solid metal circular region of radius $Q \mathrm{~cm}$ from a solid metal circular region of radius $R \mathrm{~cm}$, is below. The circles have the same center.


The ring expands as the temperature gets hotter, and so $R$ and $Q$ are each functions of the time, $t$, measured in minutes since Mitch put the ring into the oven. The following table gives some values for the functions $R$ and $Q$, as well as their derivatives.

| $t$ | 19 | 20 | 21 |
| ---: | ---: | ---: | ---: |
| $R(t)$ | 1.95 | 2 | 2.06 |
| $Q(t)$ | 1.8 | 1.75 | 1.68 |
| $R^{\prime}(t)$ | .04 | .05 | .05 |
| $Q^{\prime}(t)$ | -.06 | -.06 | -.04 |

a. [2 points] Assuming that $R(t)$ is an invertible function, compute

$$
\left(R^{-1}\right)^{\prime}(2.06)
$$

Do not give an approximation.
b. [2 points] Compute the exact value of

$$
\int_{19}^{21} Q^{\prime}(t) d t
$$

Do not give an approximation.
8. (continued)

The figure and table below are reproduced from the previous page, in case you need them on this page.


| $t$ | 19 | 20 | 21 |
| ---: | ---: | ---: | ---: |
| $R(t)$ | 1.95 | 2 | 2.06 |
| $Q(t)$ | 1.8 | 1.75 | 1.68 |
| $R^{\prime}(t)$ | .04 | .05 | .05 |
| $Q^{\prime}(t)$ | -.06 | -.06 | -.04 |

c. [2 points] Write an expression for $A(t)$, the area of the ring $t$ minutes after Mitch put it in the oven, in terms of $R(t)$ and $Q(t)$.
d. [6 points] How fast is the area of the ring growing 20 minutes after Mitch puts the ring in the oven? Include units in your answer.

