1. [9 points] Let $g(x) = x + ke^x$, where k is any constant.

a. [4 points] Write an explicit expression for the limit definition for the derivative of $g(x)$ at $x = 2$. Your expression should not include the letter 'g'. Do not evaluate your expression.

Solution:

$$g'(2) = \lim_{h \to 0} \frac{(2 + h + ke^{2+h}) - (2 + ke^2)}{h}$$

b. [5 points] Find all values of k for which the function $g(x)$ has a critical point. Do not try to use your answer from (a).

Solution: The function $g(x)$ has a critical point $\boxed{\text{whenever } k < 0.}$ Since $g'(x) = 1 + ke^x$, the only possible critical point is when $1 + ke^x = 0$, or

$$e^x = \frac{-1}{k}.$$

This equation will have no solutions if $k \geq 0$, but will have a solution for any value of k which is negative.

2. [5 points] A piece of wire of length L is cut into two pieces. One piece of length x cm is made into a circle and the rest is made into a square. Write an expression for the sum of the areas, A, of the circle and square in terms of the length L and the variable x. Do not optimize A.

Solution: The part of the wire that will be made into a circle has length x, so the circumference of that circle will be x. This means that $x = 2\pi r$, where r denotes the radius of the circle, so $r = \frac{x}{2\pi}$. The area of the circle will be $\pi r^2 = \pi \left(\frac{x}{2\pi}\right)^2$.
The part of the wire that will be made into a square has length $L - x$, so the perimeter of that square will be $L - x$. This means that $L - x = 4s$, where s denotes the length of a side of the square, so $s = \frac{L-x}{4}$. The area of the square will be $s^2 = \left(\frac{L-x}{4}\right)^2$. Thus the final answer is

$$A = \pi \left(\frac{x}{2\pi}\right)^2 + \left(\frac{L - x}{4}\right)^2$$