3. [16 points] Eddie and Laura have signed an exclusive contract to begin producing the world's first caffeinated soup, called Minestromnia. If they charge $4.00 per liter or more for the soup, then nobody will buy it. Otherwise, if they charge p dollars per liter for the soup, they will sell $g(p)$ liters, where

$$g(p) = 500(16 - p^2).$$

a. [3 points] Write an expression for the revenue $R(p)$ that Eddie and Laura will generate if they charge p dollars per liter of soup.

> *Solution:* The revenue is the price times the number of liters sold, so
>
> $$R(p) = pg(p) = 500p(16 - p^2)$$
>
> (To be really precise, we should say
>
> $$R(p) = \begin{cases} 500p(16 - p^2), & 0 \le p < 4 \\ 0, & p \ge 4 \end{cases}$$
>
> or something like this.)

b. [3 points] The ingredients in a liter of Minestromnia cost $1.00. To start their business, Eddie and Laura need to purchase a very large soup kettle and other equipment at a total cost of $700.00. Write an expression for the total cost $C(p)$, including fixed costs, of producing $g(p)$ liters of soup.

> *Solution:* The fixed costs are 700 and each liter costs a dollar, for a total cost of
>
> $$C(p) = 700 + g(p) = 700 + 500(16 - p^2).$$

c. [6 points] What price should Eddie and Laura charge per liter of Minestromnia in order to maximize their profits? Be sure to explain how you know that this price produces the maximum possible profit.

> *Solution:* Write $\pi(p) = R(p) - C(p)$. Then after some algebra, we get $\pi(p) = -500p^3 + 500p^2 + 8000p - 8700$, so
>
> $$\pi'(p) = -1500p^2 + 1000p + 8000$$
>
> Setting $\pi'(p)$ equal to zero gives a quadratic equation, whose solutions are $2\frac{2}{3} \approx 2.67$ and the illogical -2. So the critical point is $p = 2.67$. Plugging in the critical point gives
>
> $$\pi(2.67) \approx 6707.$$
>
> The reasonable prices that Eddie and Laura can set lie in a closed interval: $0 \le p \le 5$. The profit is clearly negative at both endpoints (they aren't getting any revenue at either endpoint) so the maximum profit occurs when the price is approximately $2.67.

3. (continued)

d. [4 points] Give a practical interpretation of the formula

$$g'(3.5) = -3500$$

that begins with

"If Eddie and Laura decrease the price of the soup from \$3.50 per liter to \$3.40 per liter ..."

Solution: ... they expect to sell about 350 more liters of Minestromnia.

4. [6 points] A car, initially going 100 feet per second, brakes at a constant rate (constant negative acceleration), coming to a stop in 8 seconds. Let t be the time in seconds after the car started to brake.

a. [3 points] Sketch a graph of the velocity of the car from $t = 0$ to $t = 8$, being sure to include labels.
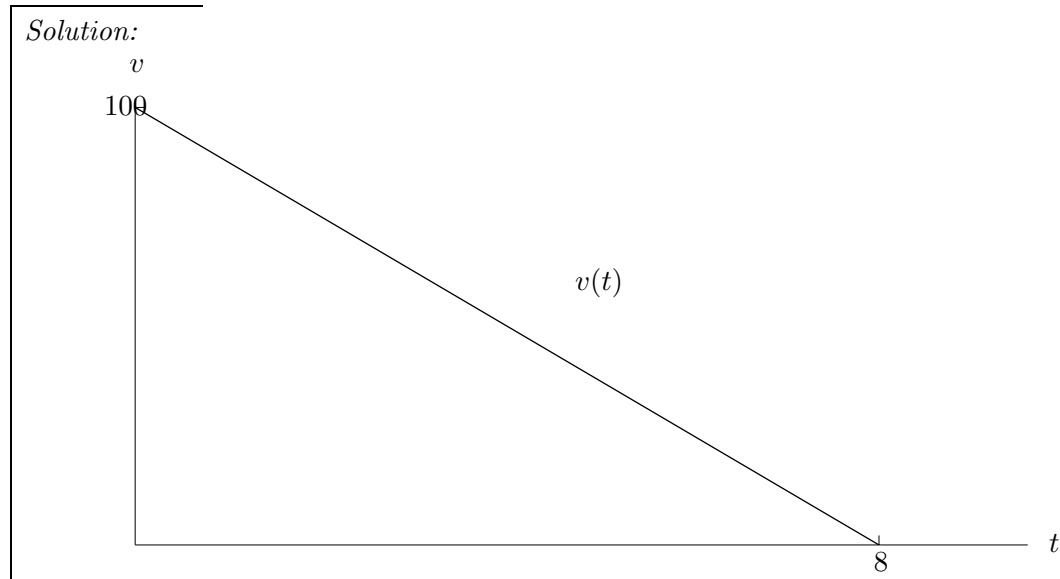
Solution:

b. [3 points] Exactly how far does the car travel? Make it clear how you obtained your answer.

Solution: It's $\int_0^8 v(t)dt$, which is the area of the triangle above: $\frac{1}{2} * 100 * 8 = 400$ ft.