5. [15 points] The function $h(x)$ is not known, but the *derivative* of $h(x)$ is given by the formula

$$h'(x) = \sin(x)e^{x^2+1}.$$

a. [2 points] Find a formula for $h''(x)$.

Solution:

$$h'(x) = \cos(x)e^{x^2+1} + 2x\sin(x)e^{x^2+1}$$

b. [6 points] List all critical points for $h(x)$ in the open interval $-2\pi < x < 2\pi$. For each point, use an appropriate test to determine whether it is a local maximum, local minimum, or neither.

Solution: A critical point of $h(x)$ occurs when $h'(x) = 0$. Since $e^{x^2+1} \neq 0$, this is when $\sin(x) = 0$, so $x = -\pi, 0, \pi$. Plugging these values into the second derivative gives

x	$g''(x)$
$-\pi$	$-e^{-\pi^2+1} < 0$
0	$e > 0$
π	$-e^{-\pi^2+1} < 0$

So $x = -\pi$ is a local maximum, $x = 0$ is a local minimum, and $x = \pi$ is a local maximum.

c. [2 points] For which x-value in the closed interval $\frac{\pi}{4} \leq x \leq \frac{\pi}{2}$ does $h(x)$ attain its maximum value? (Do not try to find the y-coordinate.)

Solution: The function $h'(x)$ is non-negative in this interval. Therefore $h(x)$ is increasing and attains its maximum value at the right endpoint $x = \frac{\pi}{2}$.

d. [5 points] Write out all terms for a right-hand Riemann sum with three subintervals which approximates

$$\int_0^1 \sin(x)e^{x^2+1}dx.$$

Solution:

$$\frac{1}{3}\left(\sin\left(\frac{1}{3}\right)e^{\left(\frac{1}{3}\right)^2+1} + \sin\left(\frac{2}{3}\right)e^{\left(\frac{2}{3}\right)^2+1} + \sin(1)e^2\right)$$