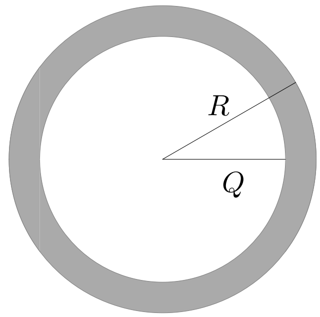
8. [12 points] Mitch puts a thin metal ring in an oven. A picture of the ring, which is made by removing a solid metal circular region of radius Q cm from a solid metal circular region of radius R cm, is below. The circles have the same center.



The ring expands as the temperature gets hotter, and so R and Q are each functions of the time, t, measured in minutes since Mitch put the ring into the oven. The following table gives some values for the functions R and Q, as well as their derivatives.

t	19	20	21
$R(t)$	1.95	2	2.06
$Q(t)$	1.8	1.75	1.68
$R'(t)$	.04	.05	.05
$Q'(t)$	−.06	−.06	−.04

a. [2 points] Assuming that $R(t)$ is an invertible function, compute

$$(R^{-1})'(2.06).$$

Do not give an approximation.

Solution: We have

$$(R^{-1})'(2.06) = \frac{1}{R'(R^{-1}(2.06))} = \frac{1}{.05} = 20 \text{ minutes/cm}$$

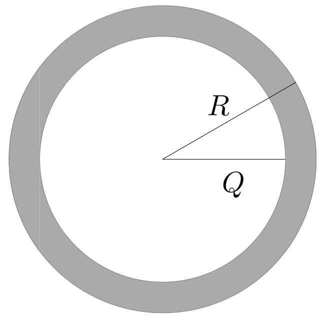
b. [2 points] Compute the exact value of

$$\int_{19}^{21} Q'(t)dt.$$

Do not give an approximation.

Solution: By the fundamental theorem of calculus $\int_{19}^{21} Q'(t)dt = Q(21) - Q(19) = -.12$

8. (continued)
The figure and table below are reproduced from the previous page, in case you need them on this page.



t	19	20	21
$R(t)$	1.95	2	2.06
$Q(t)$	1.8	1.75	1.68
$R'(t)$	.04	.05	.05
$Q'(t)$	$-.06$	$-.06$	$-.04$

c. [2 points] Write an expression for $A(t)$, the area of the ring t minutes after Mitch put it in the oven, in terms of $R(t)$ and $Q(t)$.

Solution:

$$A(t) = \pi(R(t))^2 - \pi(Q(t))^2$$

d. [6 points] How fast is the area of the ring growing 20 minutes after Mitch puts the ring in the oven? Include units in your answer.

Solution: By the chain rule,

$$A'(t) = 2\pi R(t)R'(t) - 2\pi Q(t)Q'(t),$$

so

$$A'(20) = 2\pi * 2 * .05 - 2\pi * 1.75 * (-.06) = .41\pi \frac{\text{cm}^2}{\text{min}}$$