1. [11 points] The table below gives several values of a function $f(x)$ and its derivative. Assume that both $f(x)$ and $f'(x)$ are defined and differentiable for all x.

x	0	1	2	3	4	5	6
$f(x)$	0	3	4	2	-1	-3	5
$f'(x)$	4	2	-1	-5	-2	7	9
$f''(x)$	-1	-3	-5	0	4	3	1

Compute each of the following. Do not give approximations. If it is not possible to find the value exactly, write NOT POSSIBLE

a. [2 points] Find $\int_0^4 f''(x)\,dx$.

Solution: By the Fundamental Theorem of Calculus,

$$\int_0^4 f''(x)\,dx = f'(4) - f'(0) = -2 - 4 = -6.$$

Answer: $\int_0^4 f''(x)\,dx =$ -6

b. [2 points] Find $\int_2^5 (3f(x)+1)\,dx$.

Solution: In order to evaluate this exactly, we would need to know an antiderivative of $f(x)$. Since we don't know one, this is not possible to evaluate exactly.

Answer: $\int_2^5 (3f(x)+1)\,dx =$ NOT POSSIBLE

c. [3 points] Find the average value of $4f'(x)+x$ on the interval $[1,6]$.

Solution: The average value can be computed as an integral. Since an antiderivative of $4f'(x)+x$ is $4f(x)+\frac{1}{2}x^2$, we can compute the exact value of this integral with the Fundamental Theorem of Calculus:

$$\frac{1}{6-1}\int_1^6 \left(4f'(x)+x\right)\,dx = \frac{1}{5}\left(\left(4f(6)+\frac{1}{2}6^2\right) - \left(4f(1)+\frac{1}{2}1^2\right)\right) = 5.1$$

Answer: 5.1

d. [4 points] Assuming that $f(x)$ is an odd function, find $\int_{-3}^3 f(x)\,dx$ and $\int_{-3}^3 f'(x)\,dx$.

Solution: Note that

$$\int_{-3}^3 f(x)\,dx = \int_{-3}^0 f(x)\,dx + \int_0^3 f(x)\,dx,$$

and since $f(x)$ is an odd function, the two integrals on the right cancel out, leaving us with 0.
Also, since $f(x)$ is odd, we have $f(-3) = -f(3)$, and hence by the Fundamental Theorem of Calculus,

$$\int_{-3}^3 f'(x)\,dx = f(3) - f(-3) = f(3) - (-f(3)) = 2f(3) = 4.$$

Answer: $\int_{-3}^3 f(x)\,dx =$ 0 and $\int_{-3}^3 f'(x)\,dx =$ 4