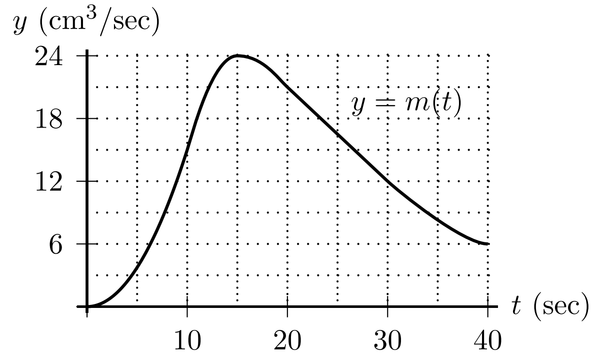
2. [10 points]
Kathy puts a very large marshmallow in the microwave for forty seconds and watches as it inflates. Let $m(t)$ be the rate of change of the volume of the marshmallow, in cm^3/sec, t seconds after Kathy puts it in the microwave. The graph of $y = m(t)$ is shown to the right.



a. [2 points] Write a definite integral equal to the total change in volume, in cm^3, of the marshmallow while in the microwave. (You do not need to evaluate the integral.)

Answer: $\int_0^{40} m(t)\,dt$

b. [3 points] Estimate your integral from part (a) using a right-hand sum with $\Delta t = 10$. Be sure to write out all of the terms in the sum.

Solution: A right-hand sum from 0 to 40 with $\Delta t = 10$ will involve the values at $t = 10$, 20, 30, and 40:

$$10m(10) + 10m(20) + 10m(30) + 10m(40) = 10(15 + 21 + 12 + 6) = 540.$$

Since $m(t)$ has units of cm^3/sec and t has units of sec, the integral has units of cm^3, which agrees with it being a change in volume.

Answer: 540 $\mathbf{cm^3}$

c. [5 points] Assume that throughout its time in the microwave, the marshmallow is a cylinder. After 30 seconds in the microwave, the marshmallow is a cylinder with radius 4.5 cm and height 11 cm. At that moment, the height is increasing at 0.08 cm/sec. How fast is the radius of the marshmallow increasing at that moment?
Recall that the volume V of a cylinder of radius r and height h is $V = \pi r^2 h$, and remember to include units.

Solution: Differentiating both sides of the volume equation with respect to t yields

$$\frac{dV}{dt} = 2\pi r h \frac{dr}{dt} + \pi r^2 \frac{dh}{dt}.$$

We are told that at this moment, $r = 4.5$, $h = 11$, and $\frac{dh}{dt} = 0.08$. Further, since $m(t) = \frac{dV}{dt}$, we can read from the graph above that at $t = 30$, we have $\frac{dV}{dt} = 12$. Plugging these in, we have

$$12 = 99\pi \frac{dr}{dt} + 1.62\pi,$$

so solving for $\frac{dr}{dt}$ yields a rate of about 0.022 cm/sec.

Answer: 0.022 **cm/sec**