4. [13 points] One of the ways Captain Christina likes to relax in her retirement is to go for long walks around her neighborhood. She has noticed that early every Tuesday morning, a truck delivers butter to a local bakery famous for its cookie dough. Consider the following functions:

- Let \( C(b) \) be the bakery’s cost, in dollars, to buy \( b \) pounds of butter.
- Let \( K(b) \) be the amount of cookie dough, in cups, the bakery makes from \( b \) pounds of butter.
- Let \( u(t) \) be the instantaneous rate, in pounds per hour, at which butter is being unloaded \( t \) hours after 4 am.

Assume that \( C, K, \) and \( u \) are invertible and differentiable.

a. [2 points] Interpret \( K(C^{-1}(10)) = 20 \) in the context of this problem.
Use a complete sentence and include units.

Solution: If the bakery spends $10 on butter, then it can make 20 cups of cookie dough.

b. [3 points] Interpret \( \int_{5}^{12} K'(b) \, db = 40 \) in the context of this problem.
Use a complete sentence and include units.

Solution: 12 pounds of butter makes 40 cups more cookie dough than 5 pounds of butter does.

c. [2 points] Give a single mathematical equality involving the derivative of \( C \) which supports the following claim:
It costs the bakery approximately $0.70 less to buy 14.8 pounds of butter than to buy 15 pounds of butter.

Answer: \( C'(15) = 3.5 \)

d. [3 points] Give a single mathematical equality which expresses the following claim:
The number of pounds of butter unloaded between 5 and 8 am is twice as many as the bakery needs to make 5000 cups of cookie dough.

Answer: \( \int_{1}^{4} u(t) \, dt = 2K^{-1}(5000) \)

e. [3 points] Assume that \( u(t) > 0 \) and \( u'(t) < 0 \) for \( 0 \leq t \leq 4 \) and that \( u(2) = 800 \).
Rank the following quantities in order from least to greatest by filling in the blanks below with the options I-IV.

I. 0 II. 800 III. \( \int_{1}^{2} u(t) \, dt \) IV. \( \int_{2}^{3} u(t) \, dt \)

Solution: Since \( u(t) > 0 \), both integrals are greater than 0. Since \( u'(t) < 0 \), \( u(t) \) is a decreasing function. Estimating \( \int_{1}^{2} u(t) \, dt \) with a right sum with one subdivision yields an underestimate of 800, and likewise, estimating \( \int_{2}^{3} u(t) \, dt \) with a left sum with one subdivision yields an overestimate of 800.

\[
0 \quad < \quad \int_{2}^{3} u(t) \, dt \quad < \quad 800 \quad < \quad \int_{1}^{2} u(t) \, dt
\]