6. [8 points] Suppose that a ring made entirely of gold and platinum is made from g ounces of gold and p ounces of platinum and that gold costs h dollars per ounce and platinum costs k dollars per ounce. Then the <u>value</u>, in dollars, of the ring is given by

$$v = gh + pk.$$

a. [3 points] Pat has a ring made entirely of gold and platinum. Pat's ring is made from 0.25 ounces of gold and 0.15 ounces of platinum. Suppose that the cost of gold is <u>decreasing</u> at an instantaneous rate of \$20 per ounce per year while the cost of platinum is <u>increasing</u> at an instantaneous rate of \$30 per ounce per year. At what instantaneous rate is the value of Pat's ring increasing or decreasing? *Remember to include units in your answer.*

> *Solution:* In this setting, g is constant at 0.25 and p is constant at 0.15, and both h and k are changing. Differentiating with respect to t, we have
>
> $$\frac{dv}{dt} = 0.25\frac{dh}{dt} + 0.15\frac{dk}{dt}.$$
>
> Plugging in $\frac{dh}{dt} = -20$ and $\frac{dk}{dt} = 30$ yields $\frac{dv}{dt} = -5 + 4.5 = -0.5$.

Answer: The value of Pat's ring is (circle one)   INCREASING   ⬚DECREASING⬚

at a rate of _____\$0.50 **per ounce per year**_____

b. [5 points] Jordan wants to design a ring made entirely of gold and platinum with a current value of \$900. Currently, gold costs \$1200 per ounce and platinum costs \$1500 per ounce. Let $w(p)$ be the total weight of Jordan's ring, in ounces, if p ounces of platinum are used.

(i) In the context of this problem, what is the domain of $w(p)$?

> *Solution:* If the ring is all gold, then we use 0 ounces of platinum. Since the ring is worth \$900, the most platinum we could possibly use is $900/1500 = 0.6$ ounces.

Answer: _____$[0, 0.6]$_____

(ii) Find a formula for $w(p)$. No variables other than p should appear in your answer.

> *Solution:* Since the value must be \$900, we have $900 = 1200g + 1500p$, or $g = 0.75 - 1.25p$. The total weight is therefore $w(p) = g + p = 0.75 - 0.25p$.

Answer: $w(p) =$ _____$0.75 - 0.25p$_____

(iii) How much gold and platinum should be in the ring if Jordan wants to <u>minimize</u> the weight of the ring? *You do not need to justify your answer.*

> *Solution:* Since $w(p)$ is linear with negative slope, the smallest value will occur when p is greatest. Therefore, it occurs at $p = 0.6$, the right endpoint of our domain, where we use 0.6 ounces of platinum and 0 ounces of gold.

Answer: _____0_____ ounces of gold and _____0.6_____ ounces of platinum