7. [7 points] The record time for the 100 meter dash is 9.58 seconds, set by Usain Bolt at a race in 2009. Let $v(t)$ be Bolt's velocity, in meters per second, t seconds after Bolt starts the race. Several values of $v(t)$ are shown below. Assume that $v(t)$ is an increasing function for the first three seconds of the race.

t	0	0.5	1	1.5	2	2.5	3
$v(t)$	0.6	3.5	5.8	7.7	9.1	10.1	10.6

(Numbers are based on data collected at the 12th IAAF World Championships in Athletics.[1]*)*

a. [2 points] Estimate Bolt's instantaneous acceleration 1.75 seconds after Bolt starts the race. *Remember to include units.*

Solution: Since acceleration is the rate of change of velocity, we estimate this with a difference quotient between $t = 1.5$ and $t = 2$:

$$a(1.75) \approx \frac{9.1 - 7.7}{2 - 1.5} = 2.8.$$

Answer: 2.8 **m/sec^2**

b. [3 points] Based on the data provided, give the best possible underestimate of the distance run by Bolt during the first 3 seconds of his race. *Be sure to show your work, and remember to include units.*

Solution: Since $v(t)$ is an increasing function, left sums provide underestimates. The best possible underestimate given the data we have, then, is a left sum with $\Delta t = 0.5$:

$$\begin{aligned} &0.5v(0) + 0.5v(0.5) + 0.5v(1) + 0.5v(1.5) + 0.5v(2) + 0.5v(2.5) \\ &\quad = 0.5(0.6 + 3.5 + 5.8 + 7.7 + 9.1 + 10.1) \\ &\quad = 18.4 \end{aligned}$$

Answer: 18.4 **m**

c. [2 points] How often would we need to measure Bolt's velocity so that the difference between the best possible underestimate and the best possible overestimate of the distance he runs in the first 3 seconds is 2 meters? *Be sure to show your work.*

Solution: If the velocity is measured in equally spaced intervals of width Δt, the difference between the left sum and right sum approximations is

$$(v(3) - v(0))\Delta t = (10.6 - 0.6)\Delta t = 10\Delta t.$$

For this to equal 2, then, we need $\Delta t = 0.2$.

Answer: Velocity must be measured every 0.2 seconds.

[1] Data available at `http://berlin.iaaf.org`