8. [11 points] A function $g(x)$ and its derivative are given by

$$g(t) = 10e^{-0.5t}(t^2 - 2t + 2) \qquad \text{and} \qquad g'(t) = -10e^{-0.5t}(0.5t^2 - 3t + 3).$$

a. [2 points] Find the t-coordinates of all critical points of $g(t)$. If there are none, write NONE. For full credit, you must find the exact t-coordinates.

Solution: Since $g'(t)$ is defined for all t, the only critical points occur where $g'(t) = 0$. To find these t values, we use the Quadratic Formula:

$$t = 3 \pm \sqrt{9-6} = 3 \pm \sqrt{3}.$$

Answer: Critical point(s) at $t =$ $3 - \sqrt{3}$, $3 + \sqrt{3}$

b. [6 points] For each of the following, find the values of t that maximize and minimize $g(t)$ on the given interval. Be sure to show enough evidence that the points you find are indeed global extrema. For each answer blank, write NONE in the answer blank if appropriate.

(i) Find the values of t that maximize and minimize $g(t)$ on the interval $[0, 8]$.

Solution: Since $[0, 8]$ is a closed interval, by the Extreme Value Theorem, $g(t)$ must have both a global max and a global min, occuring either at a critical point or at an endpoint. We therefore make a table of values at $t = 0, t = 3 - \sqrt{3} \approx 1.27$, $t = 3 + \sqrt{3} \approx 4.73$, and $t = 8$:

t	0	1.27	4.73	8
$g(t)$	20	5.69	14.01	9.16

From the table, we see the global max at $t = 0$ and the global min at $t = 3 - \sqrt{3} \approx 1.27$.

Answer: Global max(es) at $t =$ 0 Global min(s) at $t =$ $3 - \sqrt{3}$

(ii) Find the values of t that maximize and minimize $g(t)$ on the interval $[4, \infty)$.

Solution: Note that only one of our critical points, $t = 3 + \sqrt{3} \approx 4.73$, lies in this interval. Global extrema, if they exist, then, can only occur at $t = 4$ and $t = 3 + \sqrt{3}$, so we make a table of these values:

t	4	4.73
$g(t)$	13.53	14.01

We must also consider the behavior as $t \to \infty$, the open endpoint of our interval. Note that $g'(t) < 0$ for $t > 3 + \sqrt{3}$, so $g(t)$ is decreasing for $t > 3 + \sqrt{3}$. So the global max occurs at the largest value in the table, at $t = 3 + \sqrt{3} \approx 4.73$. As t gets larger and larger, $g(t) = \frac{10(t^2-2t+2)}{e^{0.5t}}$ tends to 0, as $e^{0.5t}$ grows faster than any polynomial in the long run. Since this limiting value of 0 is smaller than every value in our table, there is no global min.

Answer: Global max(es) at $t =$ $3 + \sqrt{3}$ Global min(s) at $t =$ NONE

c. [3 points] Let $G(t)$ be the antiderivative of $g(t)$ with $G(0) = -5$. Find the t-coordinates of all critical points and inflection points of $G(t)$. For each answer black, write NONE if appropriate. You do not need to justify your answers.

Solution: Critical points of $G(t)$ are zeros of $g(t)$, of which there are none. Inflection points of $G(t)$ are local extrema of $g(t)$, which occur at $t = 3 \pm \sqrt{3}$ (which we know to be local extrema because they are in fact global extrema in the interiors of some intervals).

Answer: Critical point(s) at $t =$ NONE

Answer: Inflection point(s) at $t =$ $3 - \sqrt{3}$, $3 + \sqrt{3}$