2. [13 points] For nonzero constants a and b with $b > 0$, consider the family of functions given by

$$f(x) = e^{ax} - bx\,.$$

Note that the derivative and second derivative of $f(x)$ are given by

$$f'(x) = ae^{ax} - b \qquad \text{and} \qquad f''(x) = a^2 e^{ax}\,.$$

a. [6 points] Suppose the values of a and b are such that $f(x)$ has at least one critical point. For the domain $(-\infty, \infty)$, find all critical points of $f(x)$, all values of x at which $f(x)$ has a local extremum, and all values of x at which $f(x)$ has an inflection point.
Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema and inflection points.
(Note that your answer(s) may involve the constants a and/or b.)

(For each answer blank below, write NONE in the answer blank if appropriate.)

critical point(s) at $x =$ _____ local min(s) at $x =$ _____

inflection point(s) at $x =$ _____ local max(es) at $x =$ _____

b. [2 points] Which of the following conditions on the constant a guarantee(s) that $f(x)$ has at least one critical point in its domain $(-\infty, \infty)$? Circle <u>all</u> the cases in which $f(x)$ definitely has at least one critical point. *Hint: There is at least one such condition listed.*

i. $a < 0$ ii. $0 < a < b$ iii. $b < a$

c. [5 points] Find exact values of a and b so that $f(x)$ has a critical point at $(1, 0)$.
Remember to show your work carefully.

Answer: $a =$ _____ and $b =$ _____