3. [14 points] Let g be a differentiable function defined for all real numbers. A table of some values of g is given below.

w	-1	1	3	5
$g(w)$	-2	3	5	6

Assume that g is always strictly increasing on the interval $[-1,5]$ and that g' is always strictly decreasing on the interval $[-1,5]$.

a. [2 points] Estimate $g'(5)$.

Solution: $g'(5) \approx \frac{g(5)-g(3)}{5-3} = \frac{1}{2}$.

Answer: $g'(5) \approx$ $\frac{1}{2}$

b. [4 points] Rank the following quantities in order from least to greatest by filling in the blanks below with the options I-V.

I. 0 II. $g'(1)$ III. $g(1)-g(-1)$ IV. $g'(3)$ V. $\dfrac{g(3)-g(1)}{2}$

0 $<$ $g'(3)$ $<$ $\dfrac{g(3)-g(1)}{2}$ $<$ $g'(1)$ $<$ $g(1)-g(-1)$

c. [4 points] Find the best possible estimate of $\int_{-1}^{5}(g(w)+1)\,dw$ using a right hand sum and the data provided. Be sure to write all of the terms in the sum.

Solution:

$$\begin{aligned}\int_{-1}^{5}(g(w)+1)\,dw &\approx \Delta w((g(1)+1)+(g(3)+1)+(g(5)+1))\\ &= 2(4+6+7)\\ &= 34.\end{aligned}$$

d. [1 point] Is your estimate from part (c) an overestimate or underestimate of $\int_{-1}^{5}(g(w)+1)\,dw$? *You do not need to explain your answer.*

Underestimate $\boxed{\text{Overestimate}}$ Impossible to determine

Solution: The function $g(w)+1$ is always increasing (since it is a vertical shift of $g(w)$, which is always increasing) so the right hand sum gives an overestimate.

e. [3 points] Find the average value of $g'(w)$ on the interval $[-1,5]$.

Solution: By definition, the average value of $g'(w)$ on $[-1,5]$ is

$$\begin{aligned}g'(w) &= \frac{1}{6}\int_{-1}^{5}(g'(w))\,dw\\ &= \frac{1}{6}[g(5)-g(-1)]\\ &= \frac{8}{6} = \frac{4}{3}.\end{aligned}$$

(The average value of g' on the interval is the average rate of change of g over the interval.)

Answer: $\frac{4}{3}$