5. [5 points] After hearing of the *Illumisqati* activities from Erin and Elphaba, the Police storm the King's farmhouse and find ample evidence to convict him of kidnapping. However, since he is the King, charges can only be brought against him if the Police can show proficiency in mathematics. Help them by doing the following problem.
For c a constant, consider the function $B(u) = \arctan(u^c + 7)$.
Use the limit definition of the derivative to write an explicit expression for $B'(3)$.
Your answer should not involve the letter B. Do not attempt to evaluate or simplify the limit.

Answer: $B'(3) = \boxed{\lim_{h\to 0} \frac{\arctan((3+h)^c+7) - \arctan(3^c+7)}{h}}$

6. [6 points] Recall the following definitions:

- A function f is *even* if $f(-x) = f(x)$ for all x in the domain of f.
- A function f is *odd* if $f(-x) = -f(x)$ for all x in the domain of f.

Compute each of the integrals below. If not enough information is provided to answer the question, write NOT ENOUGH INFORMATION.

a. [2 points] Suppose g is a differentiable function on $(-\infty, \infty)$ and g' (the **derivative** of g) is a continuous odd function with $g(3) = 2$ and $g(7) = 9$. Find $\int_{-3}^{7} g'(x)\,dx$.

Solution: Since $g'(x)$ is odd, $\int_{-3}^{3} g'(x)\,dx = 0$ so we have

$$\int_{-3}^{7} g'(x)\,dx = \int_{3}^{7} g'(x)\,dx = g(7) - g(3) = 9 - 2 = 7.$$

Answer: $\int_{-3}^{7} g'(x)\,dx =$ 7

b. [2 points] Suppose that q is a continuous and even function on $(-\infty, \infty)$ and that $\int_{0}^{5} q(x)\,dx = -4$. Find $\int_{-5}^{5} (3q(x) + 7)\,dx$.

Solution: By the linearity properties of definite integrals,

$$\int_{-5}^{5} (3q(x)+7)\,dx = 3\left(\int_{-5}^{5} q(x)\,dx\right) + \int_{-5}^{5} 7\,dx = 3\left(\int_{-5}^{5} q(x)\,dx\right) + 70.$$

Since $q(x)$ is even, $\int_{-5}^{5} q(x)\,dx = 2\int_{0}^{5} q(x)\,dx = -8$.

Therefore, $\int_{-5}^{5} (3q(x)+7)\,dx = 3(-8) + 70 = 46$.

Answer: $\int_{-5}^{5} (3q(x)+7)\,dx =$ 46

c. [2 points] Let $h(x) = \ln x$ and suppose p is a differentiable function on $(-\infty, \infty)$ with $p(4) = 7$. Find $\int_{4}^{1} (h(x)p'(x) + h'(x)p(x))\,dx$.

Solution: Note that $h(x)p(x)$ is an antiderivative of $h(x)p'(x) + h'(x)p(x)$ so by the Fundamental Theorem of Calculus,

$$\int_{4}^{1} (h(x)p'(x) + h'(x)p(x))\,dx = h(1)p(1) - h(4)p(4) = \ln(1)p(1) - \ln(4)p(4) = -7\ln(4).$$

Answer: $\int_{4}^{1} (h(x)p'(x) + h'(x)p(x))\,dx =$ $-7\ln(4)$