

8. [11 points] Public opinion has swung against the King since his arrest. Elphaba has been travelling the Sovereign lands collecting donations of acorns to help launch an attack against the King. Let  $P(x)$  be the total mass (in kg) of acorns that Elphaba has collected after she has travelled a total of  $x$  km. Let  $Q(t)$  be Elphaba's velocity (in km/day) when she has been travelling for  $t$  days. You may assume that  $Q(t)$  is continuous and always positive and that  $P(x)$  is an increasing, differentiable function.

For each of questions (a) through (d) below, circle the one best answer. No points will be given for ambiguous or multiple answers.

- a. [2 points] Circle the one equation below that best supports the following statement:  
*When Elphaba has travelled 100 km, she has collected approximately 3 kg less acorns than she will have collected when she has travelled 100.5 km.*

i.  $P'(100) = 6$

iv.  $P'(100.5) = -6$

ii.  $P'(100) = -3$

v.  $P'(100.5) = 3$

iii.  $P'(100) = 1.5$

vi.  $P'(100.5) = -1.5$

- b. [2 points] Which one of the following expressions is equal to the amount (in kg) by which Elphaba's collection of acorns increases over the course of the 50th km of her travels?

i.  $P(50)$

iii.  $\int_{49}^{50} P(t) dt$

ii.  $P'(49)$

iv.  $\int_{49}^{50} P'(x) dx$

- c. [2 points] Which one of the following expressions is equal to the mass (in kg) of acorns that Elphaba collected during the 4th day of her travels?

i.  $P'(4)$

iii.  $P(4) - P(3)$

ii.  $P\left(\int_0^4 Q(t) dt\right) - P\left(\int_0^3 Q(t) dt\right)$

iv.  $P\left(\int_3^4 Q(t) dt\right)$

- d. [2 points] Let  $m$  be a positive constant and let  $R(t)$  be the antiderivative of  $Q(t)$  such that  $R(0) = 0$ . Assuming that both  $P(t)$  and  $R(t)$  are invertible, which one of the following expressions is equal to the time (in days) it takes Elphaba to collect  $m$  kg of acorns?

i.  $R(P(m))$

iv.  $P(R(m))$

ii.  $R^{-1}(P^{-1}(m))$

v.  $P^{-1}(R^{-1}(m))$

iii.  $R(P^{-1}(m))$

vi.  $P(R^{-1}(m))$

- e. [3 points] Write an equation that expresses the following statement:

*After Elphaba has been travelling for a total of 5 days, she has collected a total of 200 kg of acorns.*

**Answer:** 
$$P\left(\int_0^5 Q(t) dt\right) = 200 \text{ or } \int_0^5 Q(t) dt = P^{-1}(200)$$