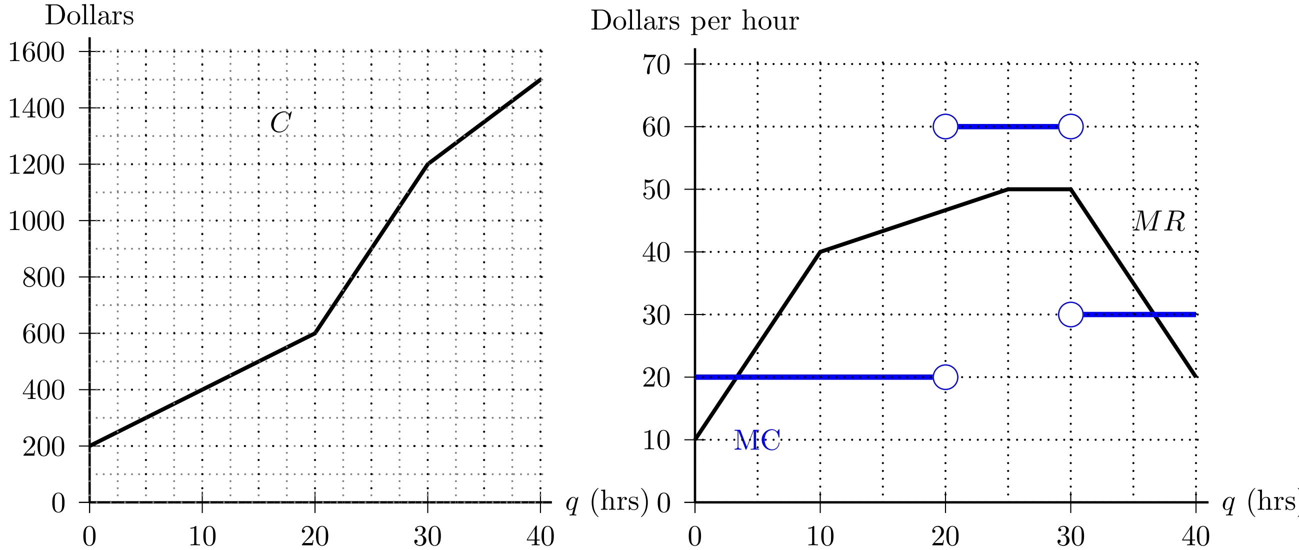
9. [10 points] Months later, the now infamous Roderick has been dethroned. Before Erin returns to the University of Michigan, she visits Roderick to hear his side of the story. He encourages her to share his story. Erin is in fact quite a good storyteller, so she begins to consider a career as a travelling storyteller. She decides to charge clients for her time (in hours).

a. [3 points] Shown below are graphs of the cost, C, and marginal revenue, MR, of Erin's potential storytelling business. Note that both graphs are continuous and piecewise linear.

Carefully sketch the graph of Erin's marginal cost function on the same axes as the given graph of her marginal revenue. (That is, draw the graph of marginal cost on the set of axes on the right.)



b. [3 points] Let $\pi(q)$ be Erin's profit from q hours of work as a travelling storyteller. Estimate all the critical points of $\pi(q)$ for $0 < q < 40$.

Solution: To find critical points of $\pi(q)$ we (i) look for points where MC=MR and (ii) look for points where $\pi'(q) = \text{MC} - \text{MR}$ is undefined. The marginal cost equals marginal revenue when $q = \frac{10}{3}$ and $q = \frac{110}{3}$ and the derivative $\pi'(q)$ is undefined when $q = 20$ and $q = 30$.

Answer: critical point(s) at $q =$ $\frac{10}{3}$, 20, 30, $\frac{110}{3}$

c. [4 points] If she can spend at most 40 hours on this venture, how many hours of work as a travelling storyteller should Erin do in order to maximize her profit? What is her maximum possible profit (in dollars)? (Assume that her revenue is 0 if she spends 0 hours storytelling.) *Briefly indicate your reasoning.*

Solution: By the extreme value theorem, since profit is continuous on the closed interval, maximum profit occurs and must occur at a critical point or endpoint of the interval. Since marginal profit is negative just before $q = 10/3$, $q = 30$, and $q = 40$, none of these can result in max profit. The area between the MR and MC curves between $q = 20$ and $q = 30$ (when $MR < MC$) is larger than that between $q = 30$ and $q = 110/3$ (when $MR > MC$), so profit decreases between $q = 20$ and $q = 110/3$. Hence max profit occurs at either $q = 20$ or $q = 0$. Similarly, comparing the areas between $q = 0$ and $q = 10$ to that between $q = 10/3$ and $q = 20$ we see that profit is greater at $q = 20$ than at $q = 0$, and max profit occurs at $q = 20$.

Answer: Maximum profit occurs at $q =$ 20

Maximum profit: $83 + \frac{1}{3}$ **dollars**