**2.** [8 points] Due to an accident, an oil pipeline is leaking.
Let $p(t)$ be the rate (in gallons/hour) at which the pipeline leaks oil $t$ hours after the accident. Assume that $p(t)$ is a strictly decreasing, differentiable function for $0 \le t \le 24$.
Engineers make the following measurements of $p(t)$.

| $t$ | 0 | 6 | 12 | 18 | 24 |
|---|---|---|---|---|---|
| $p(t)$ | 97 | 86 | 79 | 61 | 49 |

**a.** [2 points] Use the data provided, to estimate $p'(15)$. *Remember to show your work clearly.*

*Solution:*

$$p'(15) \approx \frac{p(18) - p(12)}{18 - 12} = \frac{61 - 79}{18 - 12} = -3$$

**Answer:** $p'(15) \approx$ $-3$

**b.** [3 points] Based on the data provided, write the right Riemann sum that best approximates the total amount of oil (in gallons) that leaked from the pipeline in the first 24 hours after the accident. *Be sure to carefully write out all of the terms in the sum.*

*Solution:* We want to estimate $\int_0^{24} p(t)\,dt$. Based on the limited data provided, the best we can do is to use 4 equal subintervals ($n = 4$, $\Delta t = 6$). The resulting approximation of the total number of gallons of oil that leaked from the pipeline in the first 24 hours after the accident is then

$$\int_0^{24} p(t)\,dt \approx p(6)\cdot 6 + p(12)\cdot 6 + p(18)\cdot 6 + p(24)\cdot 6 = 86(6) + 79(6) + 61(6) + 49(6) = 1650.$$

**c.** [1 point] Indicate whether the right sum above is an overestimate or an underestimate for the total amount of oil leaked. If there is not enough information to make this determination, circle "not enough information". You do not need to explain your answer.

**Answer:** The right sum is an (circle one):

overestimate $\boxed{\text{underestimate}}$ not enough information

**d.** [2 points] Suppose that engineers measured $p(t)$ every 30 minutes, starting when the pipeline started leaking, for 24 hours. What would the difference (in gallons) be between the resulting best left sum and right sum estimates for the total amount of oil leaked in those first 24 hours?

*Solution:* The difference between the left and right Riemann sums for equal subintervals with $\Delta t = 0.5$ is

$$0.5\,(p(0) - p(24)) = 0.5(97 - 49) = 24.$$

**Answer:** $0.5(97 - 49) = 24$ **gallons**