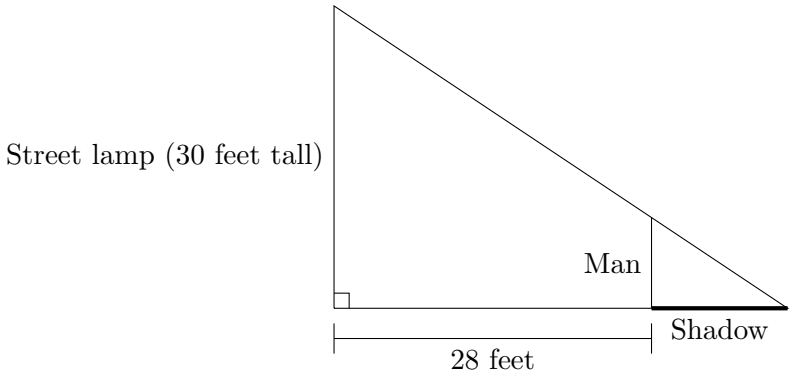
3. [8 points] A man, who is 28 feet away from a 30 foot tall street lamp, is sinking into quicksand. (*See diagram below.*) At the moment when 6 feet of him are above the ground, his height above the ground is shrinking at a rate of 2 feet/second.



Throughout this problem, remember to show your work clearly, and include units in your answers.

a. [3 points] How long will the man's shadow (shown in bold in the diagram above) be at the moment when 6 feet of him are above the ground?

Solution: Let s be the length of the shadow. Noticing that the larger and smaller triangles in the picture are similar triangles, we have

$$\begin{aligned} \frac{30}{28+s} &= \frac{6}{s} \\ 30s &= 168 + 6s \\ 24s &= 168 \\ s &= 7. \end{aligned}$$

So the length of the shadow is 7 feet at that moment.

Answer: **7 feet**

b. [5 points] At what rate is the length of the man's shadow changing at the moment 6 feet of him are above the ground? Is his shadow growing or shrinking at that moment?

Solution: Let h be the height of the man above the ground, and let s be the length of his shadow. Using similar triangles as above, we have $\frac{30}{28+s} = \frac{h}{s}$ so $30s = 28h + hs$.

Taking derivatives with respect to time t, we find $30\frac{ds}{dt} = 28\frac{dh}{dt} + h\frac{ds}{dt} + s\frac{dh}{dt}$.

So at the moment when $h = 6$, we have

$$\begin{aligned} 30\left.\frac{ds}{dt}\right|_{h=6} &= 28(-2) + 6\left.\frac{ds}{dt}\right|_{h=6} + 7(-2) \\ 24\left.\frac{ds}{dt}\right|_{h=6} &= -70 \\ \left.\frac{ds}{dt}\right|_{h=6} &= \frac{-70}{24} = -\frac{35}{12} \approx -2.917 \end{aligned}$$

So at that moment, the shadow is shrinking at a rate of about 2.917 feet/second.

Answer: The man's shadow is (circle one) GROWING **SHRINKING**

at a rate of **$\frac{35}{12}$ (about 2.917) feet/second**.