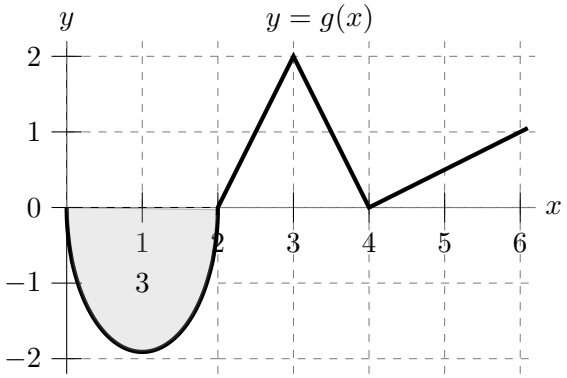
6. [10 points]

A portion of the graph of a continuous function $g(x)$ is shown on the right.
Assume that the area of the shaded region is 3 (as indicated on the graph), and note that $g(x)$ is piecewise linear for $2 < x < 6$.



For each of parts **a.-e.** below, find the value of the given quantity. If there is not enough information provided to find the value, write NOT ENOUGH INFO. If the value does not exist, write DOES NOT EXIST.

Remember to show your work.

a. [2 points] Find $\int_0^6 g(x)\,dx$.

Solution: $\int_0^6 g(x)\,dx = -3 + 2 + 1 = 0.$

Answer: 0

b. [2 points] Find $\int_0^2 (5 - 4g(x))\,dx$.

Solution: $\int_0^2 (5 - 4g(x))\,dx = \int_0^2 5\,dx - 4\int_0^2 g(x)\,dx = 5(2) - 4(-3) = 22.$

Answer: 22

c. [2 points] Suppose $C(x) = \ln(g(x))$. Find $C'(2.5)$.

Solution: $C'(x) = \dfrac{g'(x)}{g(x)}$, so $C'(2.5) = \dfrac{g'(2.5)}{g(2.5)} = \dfrac{2}{1} = 2.$

Answer: 2

d. [2 points] Find the average value of $g(x)$ on the interval $0 \le x \le 4$.

Solution: $\dfrac{1}{4-0}\int_0^4 g(x)\,dx = \dfrac{1}{4}(-3 + 2) = -\dfrac{1}{4}.$

Answer: $-\dfrac{1}{4}$

e. [2 points] Find $\int_2^4 (g(x+2) - g(x-2))\,dx$.

Solution:

$$\begin{aligned}\int_2^4 (g(x+2) - g(x-2))\,dx &= \int_2^4 g(x+2)\,dx - \int_2^4 g(x-2)\,dx \\ &= \int_4^6 g(x)\,dx - \int_0^2 g(x)\,dx \\ &= 1 - (-3) = 4\end{aligned}$$

Answer: 4