9. [8 points] Suppose g is a twice differentiable function with continuous second derivative. Several values of the first and second derivatives of g are shown in the table below.

t	-8	-6	-4	-2	0	2	4	6	8
$g'(t)$	-11	-10	-8	0	9	16	17	14	7
$g''(t)$	1	0	2	5	4	2	0	-2	-4

Assume that between each pair of consecutive values of t shown in the table, each of $g'(t)$ and $g''(t)$ is either always strictly decreasing or always strictly increasing.

a. [2 points] Use the local linearization of $g'(t)$ near $t = 6$ to estimate $g'(5.8)$.

Solution: Let $L(t)$ be the local linearization of $g'(t)$ near $t = 6$.
Then $L(t) = g'(6) + g''(6)(t - 6) = 14 - 2(t - 6)$ and we find the estimate

$$g'(5.8) \approx L(5.8) = 14 - 2(5.8 - 6) = 14 + 0.4 = 14.4.$$

Answer: $g'(5.8) \approx$ 14.4

b. [1 point] Indicate whether the local linearization of $g'(t)$ near $t = 6$ gives an overestimate or an underestimate of the value of $g'(5.8)$. If there is not enough information to make this determination, circle "not enough information". You do not need to explain.
Answer: This estimate is an (circle one):

$\boxed{\text{overestimate}}$ underestimate not enough information

c. [5 points] Let f be the quadratic function defined by $f(t) = t^2 - 4t + 6$. and let R be the function defined by $R(t) = f(t) - g(t)$. At what, if any, values of t does $R'(t)$ (the <u>derivative</u> of $R(t)$) attain its global extrema in the open interval $-8 < t < 8$?
For each answer blank, write NONE if $R'(t)$ does not attain a global extremum of that type on the open interval $-8 < t < 8$, and write NOT ENOUGH INFO if the t value(s) cannot be determined exactly. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found where the global extrema occur.

Solution: Note: Since $R(t) = f(t) - g(t) = t^2 - 4t + 6 - g(t)$, we have $R'(t) = f'(t) - g'(t) = 2t - 4 - g'(t)$ and $R''(t) = f''(t) - g''(t) = 2 - g''(t)$.
To determine the global extrema of R', we first find the critical points of R', which are the points where R'' is undefined or equal to 0. Since both f'' and g'' are defined on the entire interval $-8 < t < 8$, there are no points in the interval where R'' is undefined. Since $R''(t) = 2 - g''(t)$, we see that $R''(t) = 0$ if and only if $g''(t) = 2$. Because g'' is continuous and strictly increasing or strictly decreasing between consecutive values of t in the table, we conclude that the only values of t in the interval $-8 < t < 8$ with $g''(t) = 2$ are $t = -4$ and $t = 2$.
Note that since R' is continuous, $\lim\limits_{t \to -8^+} R'(t) = R'(-8) = -9$ and $\lim\limits_{t \to 8^-} R'(t) = R'(8) = 25$.
Also note that $R''(t) > 0$ on the intervals $(-8, -4)$ and $(2, 8)$ (since $g''(t) < 2$ on these intervals), so $R'(t)$ is strictly increasing on both of these intervals.
We consider the value of R' at its critical points and the limit of R' at the ends of the open interval.

t	$f'(t)$	$g'(t)$	$R'(t)$
$\lim\limits_{t \to -8^+}$	-20	-11	-9
$t = -4$	-12	-8	-4
$t = 2$	0	16	-16
$\lim\limits_{t \to 8^-}$	14	-11	25

We conclude that (1) the global minimum value of $R'(t)$ on the interval $-8 < t < 8$ is -16, which occurs at $t = 2$, and (2) $R'(t)$ does not attain a global maximum on the interval $-8 < t < 8$. (The value of $R'(t)$ increases arbitrarily close to 25 as t approaches 8, but there is no value of t in the open interval $-8 < t < 8$ at which the value of $R'(t)$ is actually equal to 25.)

Answer: Global min(s) at $t =$ 2 and Global max(es) at $t =$ NONE