7. [ 9 points] Consider the family of functions

$$
f(x)=a x^{2} e^{-b x}
$$

where $a$ and $b$ are positive constants. Note that

$$
f^{\prime}(x)=a x(2-b x) e^{-b x} .
$$

a. [4 points] Find the exact values of $a$ and $b$ so that $f(x)$ has a critical point at $\left(4, e^{-2}\right)$.

Answer: $a=$ $\qquad$ and $\quad b=$ $\qquad$
b. [5 points] Using your values of $a$ and $b$ from the previous part, find and classify the local extrema of $f(x)$. Use calculus to find and justify your answers, and be sure to show enough evidence that you have found them all. For each answer blank, write none if appropriate.

Answer: Local max(es) at $x=$ $\qquad$ Local $\min (\mathrm{s})$ at $x=$ $\qquad$

