3. [12 points] Virgil, Duncan, Jasper and Zander are all watching a toy wind-up mouse move across the floor. Their person places the toy on the floor 2.3 meters away from Virgil, and it moves in a straight line directly away from Virgil at a strictly decreasing velocity. Below are some values of $v(t)$, the velocity of the toy mouse, in meters per second, t seconds after the person places it on the floor, where a positive velocity corresponds to the toy moving away from Virgil.

t	0	0.25	0.5	0.75	1	1.25	1.5	1.75	2
$v(t)$	3.19	2.39	1.86	1.43	1.11	0.86	0.54	0.42	0.11

a. [4 points] Estimate the value of $\int_{0.25}^{1.75} v(t)\,dt$ using a left-hand Riemann sum with $\Delta t = 0.5$. Be sure to write down all the terms in your sum. Is your answer an over- or underestimate?

Solution: Left hand sum=$0.5(2.39 + 1.43 + 0.86) = 2.34$.

This is (circle one):

[✓ AN OVERESTIMATE] AN UNDERESTIMATE NOT ENOUGH INFORMATION

b. [3 points] How often should the values of $v(t)$ be measured in order to find upper and lower estimates for $\int_{0.25}^{1.75} v(t)\,dt$ that are within 0.1 m of the actual value?

Solution: We can estimate the size of Δt using the formula

$$|v(1.75) - v(0.25)|\Delta t = |0.42 - 2.39|\Delta t = 1.97\Delta t \leq 0.1.$$

This yields $\Delta t \leq \frac{0.1}{0.97} \approx 0.0507$ seconds.

c. [2 points] Find the value of $\int_{0.5}^{1.25} v'(t)\,dt$.

Solution: Using the Fundamental Theorem of Calculus $\int_{0.5}^{1.25} v'(t)\,dt = v(1.25) - v(0.5) = 0.86 - 1.86 = -1$.

d. [3 points] Which of the following represents how much the distance from the toy mouse to Virgil increases during the 2^{nd} second after it has been placed on the floor? Circle the one best answer.

i. $2.3 - \int_1^2 v(t)\,dt$

ii. $2.3 - \int_1^2 v'(t)\,dt$

iii. $\int_1^2 v(t)dt - \int_0^1 v(t)\,dt$

[✓ iv. $\int_1^2 v(t)\,dt$]

v. $\int_1^2 v'(t)\,dt$

vi. $v(2) - v(1)$

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