

4. [5 points] Consider the function $Z(w) = \arctan(kw) - (w + 1)$ where k is a nonzero constant. Use the limit definition of the derivative to write an explicit expression for $Z'(-2)$. *Your answer should not involve the letter Z . Do not attempt to evaluate or simplify the limit.* Please write your final answer in the answer box provided below.

Solution:

Answer: $Z'(-2) = \lim_{h \rightarrow 0} \frac{\arctan(k(-2+h)) - ((-2+h) + 1) - (\arctan(-2k) + 1)}{h}$

5. [9 points] A cylindrical bar of radius R and length L (both in meters) is put into an oven. As the bar gains temperature, its radius decreases at a constant rate of 0.05 meters per hour and its length increases at a constant rate of 0.12 meters per hour. Fifteen minutes after the bar was put into the oven, its radius and length are 0.4 and 3 meters respectively. At what rate is the volume of the bar changing at that point? *Be sure to include units.*

Solution: The volume of the cylindrical bar is $V = \pi R^2 L$. Differentiating with respect to t , we obtain

$$\frac{dV}{dt} = \pi \left(2R \frac{dR}{dt} L + R^2 \frac{dL}{dt} \right).$$

You are given that after 15 minutes: $R = 0.4$, $L = 3$, $\frac{dR}{dt} = -0.05$ and $\frac{dL}{dt} = 0.12$. Then

$$\frac{dV}{dt} = \pi (2(0.4)(-0.05)(3) + (0.4)^2(0.12)) = -0.3166.$$

Answer: The volume of the bar is (circle one):

INCREASING

DECREASING

NOT ENOUGH INFORMATION

at a rate of 0.3166 m³ per hour.