8. [16 points] An apple farmer starts harvesting apples on her orchard. They start collecting apples at 6 am. Let $a(t)$ be the total amount of apples (in hundreds of pounds) that have been harvest t hours after 6 am. Some of the values of the invertible function $a(t)$, its derivative $a'(t)$ and an antiderivative function $b(t)$ are shown below.

t	3	4.5	6	7.5	9	10.5	12
$a(t)$	1.5	2	3	4.5	6	6.5	9

t	3	6	9	12
$a'(t)$	0.4	1.2	0.5	1.8

t	3	6	9	12
$b(t)$	6	12.5	25.5	43

a. [2 points] Use the tables to estimate the value of $a''(9)$. Show your work.

Solution: Possible approximations:
$a''(9) \approx \dfrac{1.8-0.5}{12-9} \approx 0.433$, $a''(9) \approx \dfrac{0.5-1.2}{9-6} \approx -.233$ or $a''(9) \approx \dfrac{0.433-.233}{2} = 0.1$

b. [3 points] Find the value of $(a^{-1})'(6)$. What are its units in the context of this problem?

Solution: $(a^{-1})'(6) = \dfrac{1}{a'(a^{-1}(6))} = \dfrac{1}{a'(9)} = \dfrac{1}{0.5} = 2$ hours per hundreds of pounds of apples.

c. [3 points] Use the fact that $a'(10) = 3.2$ to complete the sentence below, including units, to give a practical interpretation in the context of this problem that can be understood by someone who knows no calculus.
The amount of apples harvested between 4 pm and 4:30 pm ...

Solution: increases by approximately 160 pounds of apples.

d. [3 points] Find the tangent line approximation $S(t)$ of $b(t)$ near $t = 3$.

Solution: $S(t) = b(3) + b'(3)(t-3) = 6 + 1.5(t-3)$.

e. [2 points] Use your answer in **d** to approximate the value of $b(2)$.

Solution: $b(2) \approx S(2) = 6 - 1.5 = 4.5$.

f. [1 point] Is your answer in **e** an overestimate or an underestimate? Circle your answer.

Solution:

OVERESTIMATE $\boxed{\text{UNDERESTIMATE}}$ NOT ENOUGH INFO

g. [2 points] Let $m(t)$ be an antiderivative of $a(t)$ satisfying $m(9) = -1$. Find $m(3)$.

Solution: We know that two antiderivatives $b(t)$ and $m(t)$ of $a(t)$ satisfy $m(t) = b(t) + C$. Then using $t = 9$ we get that $C = m(9) - b(9) = -1 - 25.5 = -26.5$. Hence $m(3) = b(3) - 26.5 = 6 - 26.5 = -20.5$.