1. [13 points] The function $P(x)$ is defined on the interval $-14 \leq x \leq 14$. The graph of $P(x)$ is shown below for $0 \leq x \leq 10$.

The function $P(x)$ has the following properties:

- it is an even function,
- the shaded region has area equal to 3,
- $P(x)$ is twice differentiable on $(9, 14)$ and $P$, $P'$, and $P''$ have the following values

<table>
<thead>
<tr>
<th>$x$</th>
<th>10</th>
<th>11</th>
<th>12</th>
<th>13</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P(x)$</td>
<td>2</td>
<td>2.5</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>$P'(x)$</td>
<td>-0.5</td>
<td>0.2</td>
<td>-2</td>
<td>1.5</td>
</tr>
<tr>
<td>$P''(x)$</td>
<td>0</td>
<td>-0.5</td>
<td>1.7</td>
<td>2.5</td>
</tr>
</tbody>
</table>

In the following questions, your answers must be exact. If any of the answers are undefined, write “UND”. If there is not enough information to answer a question, write “NEI”.

a. [2 points] Find $\lim_{m \to 0} \frac{P(m+12) - P(12)}{m}$.

**Solution:** $P'(12) = -2$  
**Answer:** $-2$

b. [2 points] Let $J(x)$ be an antiderivative of $P(x)$. Find $J'(3)$.

**Solution:** $J'(3) = P(3) = -3$.  
**Answer:** $-3$.

c. [2 points] Let $K(x)$ be an antiderivative of $P(x)$ with $K(8) = -2$. Find $K(0)$.

**Solution:** $K(8) - K(0) = \int_0^8 P(x)dx$ so $K(0) = -2 - 3 = -5$  
**Answer:** $-5$.

d. [3 points] Find $\int_{-3}^{6} (2P(t) + 1)dt$.

**Solution:** $\int_{-3}^{6} (2P(t) + 1)dt = 2\int_{-3}^{6} P(t)dt + 9 = 2(-6) + 9 = -3$  
**Answer:** $-3$.

e. [2 points] Find $\int_{10}^{13} P''(x)dx$.

**Solution:** $\int_{10}^{13} P''(x)dx = P'(13) - P'(10) = 1.5 - (-0.5) = 2$  
**Answer:** $2$.

f. [2 points] Let $Q(x) = P(3x^2 + 1)$. Find $Q'(2)$.

**Solution:** $Q'(x) = P'(3x^2 + 1)(6x)$ then $Q'(2) = P'(13)(12) = (1.5)(12) = 18$  
**Answer:** $18$. 

University of Michigan Department of Mathematics  
Winter, 2018 Math 115 Exam 3 Problem 1 Solution