2. [15 points] There were 3 trillion trees in the world in the year 2000.

- Since the year 2000, a group of environmentalists have recorded the number of trees lost in the world due to natural causes or due to human activities. Let $C(t)$ be the rate at which the number of trees decreases due to any of these causes, $t$ years after the year 2000, in trillions of trees per year.

- At the same time, some governments and other organizations plant new trees to increase the number of trees in the world. The group is also measuring the rate $P(t)$ at which the trees are being planted, $t$ years after the year 2000, in trillions of trees per year.

Throughout this question, you may assume that the functions $C(t)$ and $P(t)$ describe the only changes to the number of trees in the world.

a. [7 points] In parts (i) and (ii) below, give a mathematical expression that may involve $C(t)$, $P(t)$, their derivatives, and/or definite integrals.

(i) Find an expression for the total number of trees in the world (in trillions) in the year 2005.

**Solution:**

**Answer:** $3 + \int_{0}^{5} (P(t) - C(t)) \, dt$

(ii) Find an expression for the average rate at which the trees were being planted (in trillions of trees per year) between the years 2002 and 2009.

**Solution:**

**Answer:** $\frac{1}{7} \int_{2}^{9} P(t) \, dt$

b. [3 points] Write a practical interpretation of the statement $\int_{13}^{17} C(t) \, dt = 0.05$. Your answer must be a complete sentence.

**Solution:** Between 2013 and 2017, 0.05 trillion (50 billion) trees were cut.
The question has been reproduced here for your convenience.

There were 3 trillion trees in the world in the year 2000.

- Since the year 2000, a group of environmentalists have recorded the number of trees lost in the world due to natural causes or due to human activities. Let \( C(t) \) be the rate at which the number of trees decreases due to any of these causes, \( t \) years after the year 2000, in trillions of trees per year.

- At the same time, some governments and other organizations plant new trees to restore the forests. The group is also measuring the rate \( P(t) \) at which the trees are being planted, \( t \) years after the year 2000, in trillions of trees per year.

Throughout this question, you may assume that the functions \( C(t) \) and \( P(t) \) describe the only changes to the number of trees in the world.

c. [5 points] Additionally, you know that

- \( C(t) = P(t) \) in 2001 and 2009,
- between 2000 and 2010, the number of trees in the world was the largest in 2006.

The graph of \( P(t) \) is given below for \( 0 \leq t \leq 10 \). In the same axis, sketch a possible graph of \( C(t) \) that is consistent with the above information. Note that there may be many correct answers.