

2. [15 points] There were 3 trillion trees in the world in the year 2000.

- Since the year 2000, a group of environmentalists have recorded the number of trees lost in the world due to natural causes or due to human activities. Let  $C(t)$  be the rate at which the number of trees decreases due to any of these causes,  $t$  years after the year 2000, in trillions of trees per year.
- At the same time, some governments and other organizations plant new trees to increase the number of trees in the world. The group is also measuring the rate  $P(t)$  at which the trees are being planted,  $t$  years after the year 2000, in trillions of trees per year.

Throughout this question, you may assume that the functions  $C(t)$  and  $P(t)$  describe the only changes to the number of trees in the world.

a. [7 points] In parts (i) and (ii) below, give a mathematical expression that may involve  $C(t)$ ,  $P(t)$ , their derivatives, and/or definite integrals.

(i) Find an expression for the total number of trees in the world (in trillions) in the year 2005.

*Solution:*

$$\text{Answer: } 3 + \int_0^5 (P(t) - C(t)) dt$$

(ii) Find an expression for the average rate at which the trees were being planted (in trillions of trees per year) between the years 2002 and 2009.

*Solution:*

$$\text{Answer: } \frac{1}{7} \int_2^9 P(t) dt$$

b. [3 points] Write a practical interpretation of the statement  $\int_{13}^{17} C(t) dt = 0.05$ . Your answer must be a complete sentence.

*Solution:* Between 2013 and 2017, 0.05 trillion (50 billion) trees were cut.

The question has been reproduced here for your convenience.

There were 3 trillion trees in the world in the year 2000.

- Since the year 2000, a group of environmentalists have recorded the number of trees lost in the world due to natural causes or due to human activities. Let  $C(t)$  be the rate at which the number of trees decreases due to any of these causes,  $t$  years after the year 2000, in trillions of trees per year.
- At the same time, some governments and other organizations plant new trees to restore the forests. The group is also measuring the rate  $P(t)$  at which the trees are being planted,  $t$  years after the year 2000, in trillions of trees per year.

Throughout this question, you may assume that the functions  $C(t)$  and  $P(t)$  describe the only changes to the number of trees in the world.

c. [5 points] Additionally, you know that

- $C(t) = P(t)$  in 2001 and 2009,
- between 2000 and 2010, the number of trees in the world was the largest in 2006.

The graph of  $P(t)$  is given below for  $0 \leq t \leq 10$ . In the same axis, **sketch a possible graph** of  $C(t)$  that is consistent with the above information. Note that there may be many correct answers.

