3. [16 points] Last summer, Brad and Angelina set up a lemonade stand where they sold lemonade charging 60 cents per ounce. In other words, $MR(q) = 0.6$ (in dollars per ounce) where q is the number of ounces of lemonade they sold.

a. [2 points] Find a formula for their revenue $R(q)$ (in dollars) were q is the amount (in ounces) of lemonade sold. Assume their initial revenue is zero dollars.

Solution: **Answer:** $R(q) = 0.6q$.

Since they are using utensils they already have, they have no fixed costs. They can produce at most 120 ounces of lemonade, and the **marginal cost** function, $MC(q)$, is:

- continuous for $0 < q < 120$,
- concave down for $0 < q < 120$,
- increasing for $0 < q < 60$ and decreasing for $60 < q < 120$.

Brad and Angelina recorded some of the values of $MC(q)$ (in dollars per ounce) in the following table:

q	0	15	30	45	60	75	90	105	120
$MC(q)$	0.15	0.45	0.6	0.7	0.75	0.7	0.6	0.45	0.15

b. [3 points] Recall that $C(60) - C(0) = \int_0^{60} MC(q)\ dq$. Estimate $C(60)$ by using a right-hand Riemann sum with 2 equal subdivisions. Make sure to write down all terms in your sum.

Solution: **Answer:** $30(0.6 + 0.75) = 40.5$

c. [1 point] Is your estimate in **b** an overestimate or an underestimate of $C(60)$? Circle your answer.

Solution:

$\boxed{\text{OVERESTIMATE}}$ UNDERESTIMATE NOT ENOUGH INFORMATION

d. [3 points] Suppose Brad and Angelina want to use a Riemann sum to calculate $C(60)$, accurate within 50 cents of the actual value. At least how many times, using equal intervals on $[0, 60]$, should Brad and Angelina have measured $MC(q)$ in order to guarantee this accuracy? Justify your answer.

Solution:

$$\begin{aligned}\text{Difference between righthand and lefthand sums} &= (MC(60) - MC(0))\Delta q \\ &= (0.75 - 0.15)\Delta q = 0.6\Delta q \leq 0.5.\end{aligned}$$

yields $\Delta q \leq \frac{5}{6} \approx .833$. Recall that $\Delta q = \frac{b-a}{n}$ where n is the number of measurements. This gives:

$$n = \frac{60}{\Delta q} = \frac{60}{5/6} = 72.$$

Answer: They should have measured it at least 72 times.

A part of the question has been reproduced for your convenience.

Brad and Angelina have no fixed costs and they can produce at most 120 ounces of lemonade. The **marginal cost** function (in dollars per ounce), $MC(q)$, is:

- continuous for $0 < q < 120$,
- concave down for $0 < q < 120$,
- increasing for $0 < q < 60$ and decreasing for $60 < q < 120$.

and some of its values are:

q	0	15	30	45	60	75	90	105	120
$MC(q)$	0.15	0.45	0.6	0.7	0.75	0.7	0.6	0.45	0.15

The marginal revenue is $MR(q) = 0.6$ (in dollars per ounce).

e. [2 points] Find all the critical points of the profit function $\pi(q)$.

Solution: **Answer:** $q = 30, 90$.

The table below gives some values of $C(q)$ (in dollars):

q	15	30	45	60	75	90	105	120
$C(q)$	4.7	12.7	22.5	33.5	44.5	54.3	62.8	67

Use the values in the table to answer the following question.

f. [5 points] How many ounces of lemonade should Brad and Angelina sell in order to maximize their profit, and what is their maximal profit? Use calculus to fully justify that your answer is a global maximum. Remember to include units in your answer.

Solution: The critical points are $q = 30, 90$ and the end points are $q = 0$ and $q = 120$.

q	0	30	90	120
$R(q)$	0	18	54	72
$C(q)$	0	12.7	54.3	67
$\pi(q) = R(q) - C(q)$	0	5.3	-0.3	5

Answer: Brad and Angelina's profit is maximized when they sell 30 ounces

of lemonade, and their maximal profit is 5.30 dollars.