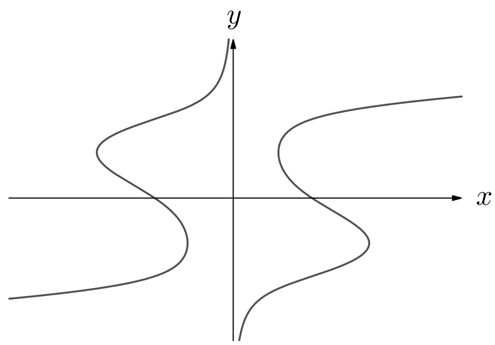
4. [8 points] Consider the curve $\mathcal{C}$ given by

$$3xy + x^2 = xy^3 + 3,$$

and note that it satisfies

$$\frac{dy}{dx} = \frac{2x - y^3 + 3y}{3xy^2 - 3x}.$$

The graph of $\mathcal{C}$ is shown below



a. [3 points] Find an equation of the tangent line to the curve $\mathcal{C}$ at the point $(3, 2)$.

Solution:

$$m = \frac{2(3) - (2)^3 + 3(2)}{3(3)(2)^2 - 3(3)} = \frac{4}{27}.$$

Answer: $y = 2 + \frac{4}{27}(x - 3)$

b. [5 points] Find the coordinates (x, y) of the point(s) with $y > 0$ at which the curve $\mathcal{C}$ has a vertical tangent line. Show all your work to justify your answer(s).

Solution: We need to solve

$$3xy^2 - 3x = 0$$
$$3x(y - 1)(y + 1) = 0.$$

yields $x = 0$ or $y = \pm 1$.
In the case of $x = 0$, the y-coordinates of the points $(0, y)$ in the curve $\mathcal{C}$ satisfy

$$3(0)y + (0)^2 = (0)y^3 + 3$$

which yields to $0 = 3$. That means that there are no points in the curve $\mathcal{C}$ with x-coordinate equal to 0.
Since we are looking for points above the x-axis ($y > 0$), then we only look for points $(x, 1)$ in the curve $\mathcal{C}$. Then the x-coordinates satisfy

$$3x(1) + x^2 = x(1)^3 + 3$$
$$x^2 - 2x - 3 = 0$$
$$(x - 1)(x + 3) = 0 \quad x = 1, -3.$$

Answer: The coordinates are $(\mathbf{1}, \mathbf{1})$ and $(\mathbf{-3}, \mathbf{1})$.