5. [6 points] There are two towns A and B that are 2 and 5 miles away from a train track and town B is 4 miles to the right of town A (see picture below). The railway company is considering building a train station in between the two towns. The new station will be built next to the train tracks and the local authorities have agreed to build roads directly connecting the two towns to the station.

The picture below shows a map of the towns and the train track along with the distances and a potential location of the station [the picture is not to scale]. Note that x denotes the distance (in miles) from the station to the point closest to Town A from the train track.



a. [3 points] Find a formula for D(x), the sum of the distances (in miles) from Town A to the station and Town B to the station if the station is x miles along the track to the right of Town A. Circle the best answer.

Solution:
(i)
$$D(x) = \sqrt{2 + x^2} + \sqrt{5 + (4 - x)^2}$$

(ii) $D(x) = \sqrt{2^2 + x^2} + \sqrt{5^2 + x^2}$
(iii) $D(x) = \sqrt{2^2 + x^2} + \sqrt{5^2 + x^2}$
(iv) $D(x) = \sqrt{2^2 + x^2} + \sqrt{5^2 + (4 - x)^2}$
(v) $D(x) = \sqrt{2 + x} + \sqrt{5 + (4 - x)^2}$
(iii) $D(x) = \sqrt{2^2 + (4 - x)^2} + \sqrt{5^2 + x^2}$
(vi) $D(x) = \sqrt{4^2 + x^2} + \sqrt{2^2 + (5 - x)^2}$

b. [3 points] The people who live in Town A negotiated a deal with the railway company that guarantees the station will be within 3 miles of their town. The railway company will build the station in between the two towns (to the right of Town A and to the left of Town B). Given this information, what is the domain for of D(x)?

Solution: The distance from Town A to the station at a distance x miles on the side of the tracks is $\sqrt{4 + x^2}$. Then we need

$$\sqrt{4+x^2} \le 3$$
$$x^2 \le 5 \qquad -\sqrt{5} \le x \le \sqrt{5}$$

Since the station is between the two towns on the side to the tracks, x cannot be negative. The domain is therefore: $0 \le x \le \sqrt{5}$.