6. [14 points] The function $f(x)$ is defined on the interval $-6<x<6$. The graphs of $f(x)$ and its derivative $f^{\prime}(x)$ are shown below on the intervals $(-6,0]$ and $(0,6)$ respectively. All the graphs consist of line segments and quarters of circles.

$$
y=f(x)
$$



$$
y=f^{\prime}(x)
$$



The function $f(x)$ is continuous at $x=0$. In the following questions, your answers must be exact. If any of the answers are undefined write "UND". If there is not enough information to answer a question, write "NEI"
a. $[2$ points $]$ Find $\lim _{x \rightarrow 4^{+}}(5 f(-x)+3)$.

Solution:
Answer: 13
b. [2 points] Find $\lim _{x \rightarrow-\infty} f\left(-4-2^{x}\right)$.

Solution:
Answer: 2
c. [2 points] On which interval(s) in $-6<x<6$ is the function $f(x)$ is decreasing?

Solution:
Answer: $[-2,1]$.
d. [3 points] At which value(s) of $-6<x<6$ is the function not differentiable?

## Solution:

Answer: $x=-4,-2,0$.
e. [3 points] Find the coordinates $(x, y)$ of the global maximum of $f(x)$ for $0 \leq x \leq 5$. Show your work.
Solution: Global maximum at $x=5$ and its $y$-coordinate is equal to

$$
f(5)=-1+\int_{0}^{5} f^{\prime}(x) d x=-1+2+\frac{1}{4} \pi(2)^{2}=1+\pi
$$

Answer: $x=5 \quad y=1+\pi$.
f. [2 points] At which value(s) of $-6<x<6$ does the function $f(x)$ have an inflection point?

Solution: The only point where the function $f(x)$ changes concavity is at $x=5$. Answer: $x=5$

