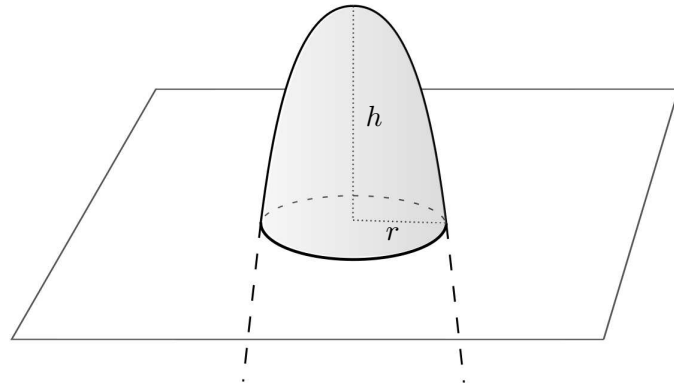


7. [11 points] A group of meteorologists observe that the sea level is rising by observing a piece of a rock in the sea.

Only the tip of the rock is visible, and as the sea water rises, less and less of the rock is above water.

Let h and r be the height and radius (in inches), respectively, of the part of the rock that is above the sea. The volume of the rock (in cubic inches) is then given by the formula

$$V = \frac{\pi}{2}(1 + r^2)h.$$



- a. [8 points] The meteorologists notice that, as the level of the sea is rising, the radius and volume of the rock are changing. A year after they started taking the measurements, the radius and height of the rock are 5 and 46 inches, respectively. They notice that at that time, the radius is decreasing at a rate of 0.05 inches per year, which makes the volume change at a rate of 80 cubic inches per year. At what rate is the height of the rock changing at that time? Be sure to include units.

Solution:

$$V = \frac{\pi}{2}(1 + r^2)h$$

$$\frac{dV}{dt} = \frac{\pi}{2} \left(2r \frac{dr}{dt} \right) h + \frac{\pi}{2}(1 + r^2) \frac{dh}{dt}$$

$$\frac{dV}{dt} = \pi r h \frac{dr}{dt} + \frac{\pi}{2}(1 + r^2) \frac{dh}{dt}$$

$$-80 = \pi(5)(46)(-0.05) + \frac{\pi}{2}(1 + (5)^2) \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{-80 + 11.5\pi}{13\pi} \approx -1.07$$

Answer: The height of the part of the rock that is above the sea is (circle one)

INCREASING

DECREASING

NOT ENOUGH INFORMATION

at a rate of 1.07 inches per year.

- b. [3 points] Meteorologists discover that the sea level increases as a function of the average temperature registered during the year. Let $R(T)$ be the sea level (in centimeters) if the average temperature next year is T degrees Celsius ($^{\circ}\text{C}$). Circle the one statement below that is best supported by the equation

$$R'(16) = 0.5.$$

- (I) The sea level will rise by about 0.5 centimeters if the average temperature next year is 16 degrees Celsius.
- (II)

The sea level will rise by approximately 0.1 centimeters if the average temperature next year rises from 15.8°C to 16°C next year.
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- (III) The average temperature next year needs to increase by about 0.5°C in order for the sea level to rise to a level of 16 centimeters next year.
- (IV) The sea level will rise by about 0.2 centimeters if the average temperature next year rises from 16°C to 16.2°C next year.
- (V) The sea level rises by 0.5 centimeters for every additional degree Celsius the average temperature rises above 16°C .