8. [6 points] A botanist is studying the number of offspring plants that will survive for a year in her experimental crop. She notices that the number of offspring depends on the number of seeds sown in the crop. Let $P(s)$ be the number of plants that will survive after a year in the crop if s thousand seeds are sown in the crop. The formula for $P'(s)$ is

$$P'(s) = \frac{1000(1-ks)(1+ks)}{(1+(ks)^2)^2} \qquad \text{for} \quad 0 \le s < \infty,$$

where $k \ge \frac{1}{2}$ is a constant.

a. [6 points] Find and classify all the critical points of $P(s)$ on $(0, \infty)$. Use calculus to find and justify your answers, and be sure to show enough evidence to demonstrate that you have found all local extrema. For each answer blank below, write "NONE" if appropriate. Your answer may depend on the constant k.

Solution: Critical points only when $P'(s) = 0$. That is when $1000(1-ks)(1+ks) = 0$. Solving for s we get $s = -\frac{1}{k}$ and $s = \frac{1}{k}$. Only $s = \frac{1}{k}$ is positive.

Classification of $s = \frac{1}{k}$:

- For $0 \le s < \frac{1}{k}$ we can pick $s = 0$ and obtain $P'(0) = 1000 > 0$.
- For $\frac{1}{k} < s$ we can pick $s = \frac{2}{k}$ and obtain $P'\left(\frac{2}{k}\right) = \frac{1000(1-2)(1+2)}{(1+(2)^2)^2} = -120 < 0$.

Therefore, $s = \frac{1}{k}$ is a local maximum.

Answer: Critical point(s) at $s = \dfrac{1}{k}$

Local maximum(s) at $s = \dfrac{1}{k}$ Local minimum(s) at $s =$ NONE