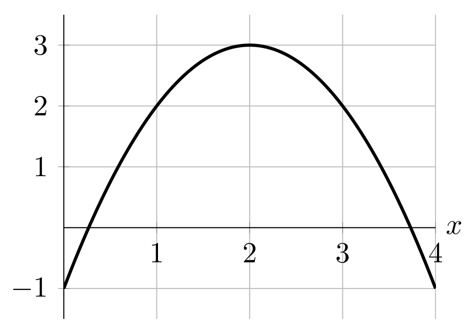
9. [11 points] Let $Q(x) = -(x-2)^2 + 3$ be the quadratic approximation of the function $y = f(x)$ at $x = 3$. A part of the graph of $Q(x)$ is shown below.



a. [6 points] If possible, find the following quantities exactly. If there is not enough information to obtain an **exact** answer, write "NEI".

Solution:

$$f''(3) = -2, \quad f'''(3) = \text{NEI}, \quad f(0) = \text{NEI},$$

$$Q''(3) = -2, \quad Q'''(3) = 0, \quad Q(0) = -1.$$

b. [4 points] Assume that the function $f(x)$ is invertible and let $g(y) = f^{-1}(y)$ be its inverse. Given that $f(3) = 2$, find the linear approximation $L(y)$ of $g(y)$ at $y = 2$. Your answer should not include the letters f or g. Show all your work.

Solution: The formula for $L(y)$ is given by

$$L(y) = f^{-1}(2) + (f^{-1})'(2)(y-2).$$

We know that $f^{-1}(2) = 3$ and $(f^{-1})'(y) = \frac{1}{f'(f^{-1}(y))}$. Hence

$$(f^{-1})'(2) = \frac{1}{f'(f^{-1}(2))} = \frac{1}{f'(3)} = -\frac{1}{2}$$

Answer: $L(y) = 3 - \frac{1}{2}(y-2)$

c. [1 point] Use the linear approximation $L(y)$ to approximate a solution to the equation $f(x) = 1.7$.

Solution:

Answer: $L(1.7) = 3 - \frac{1}{2}(1.7-2) = 3.15.$