3. [9 points] Given below is a table of values for an **odd** function $g(x)$, its derivative $g'(x)$, and its second derivative $g''(x)$. The functions $g(x)$, $g'(x)$, and $g''(x)$ are all continuous and defined for all real numbers.

x	0	4	7	9
$g(x)$	0	3	2	7
$g'(x)$	4	-1	0	-3
$g''(x)$	0	6	3	-9

Find the following values exactly, or write NEI if there is not enough information provided to do so. You do not need to show work, but limited partial credit may be awarded for work shown.

a. [2 points] $\lim_{r\to 0} \dfrac{g'(4+r)+1}{r}$

Solution: This is the limit definition of the derivative of $g'(x)$ at $x=4$. Therefore its value is $g''(4)=6$.

Answer: $=$ **6**

b. [1 point] $g'(-4)$

Solution: The derivative of an odd function is an even function. So, $g'(x)$ is even. This means that $g'(-4)=g'(4)=-1$.

Answer: $=$ -1

c. [2 points] $\displaystyle\int_0^7 \left(g'(x)+1\right)\,dx$

Solution:

$$\begin{aligned}&= \int_0^7 g'(x)\,dx + \int_0^7 1\,dx\\ &= (g(7)-g(0))+7\\ &= 2+7=9\end{aligned}$$

Answer: $=$ 9

d. [2 points] $\displaystyle\int_{-8}^{8} g(x)\,dx$

Solution: Since $g(x)$ is odd, $\int_{-a}^{a} g(x)\,dx = 0$ for any choice of a.

Answer: $=$ **0**

e. [2 points] the average value of $g''(x)$ on $[7,9]$

Solution:

$$\begin{aligned}\frac{1}{2}\int_7^9 g''(x)\,dx &= \frac{1}{2}\left(g'(9)-g'(7)\right)\\ &= \frac{-3}{2}\end{aligned}$$

Answer: $=$ $\dfrac{-3}{2}$